5 De stelling van Pythagoras

Voorkennis Kwadraten en wortels

Bladzijde 8

1 **a** 16,276 **b** 16,28 **c** 16

2 **a** 6,954 **b** 7,0 **c** 7

3
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$
$16^2 = 256$
$17^2 = 289$
$18^2 = 324$
$19^2 = 361$
$20^2 = 400$
$25^2 = 625$
$30^2 = 900$
$40^2 = 1600$
$50^2 = 2500$
$100^2 = 10\,000$

4
a $0{,}4^2 = 0{,}16$
b $200^2 = 40\,000$
c $120^2 = 14\,400$
d $1{,}2^2 = 1{,}44$
e $0{,}12^2 = 0{,}0144$
f $250^2 = 62\,500$

Bladzijde 9

5
a $\sqrt{36} = 6$
b $\sqrt{169} = 13$
c $\sqrt{49} = 7$
d $2\sqrt{121} = 2 \cdot 11 = 22$
e $\frac{1}{2}\sqrt{144} = \frac{1}{2} \cdot 12 = 6$
f $3\sqrt{16} = 3 \cdot 4 = 12$

6
a $\sqrt{8} \approx 2{,}83$
b $\sqrt{21} \approx 4{,}58$
c $\sqrt{160} \approx 12{,}65$
d $\sqrt{99} \approx 9{,}95$
e $\sqrt{50} \approx 7{,}07$
f $\sqrt{200} \approx 14{,}14$

7
a lengte zijde van vierkant $ABCD$ is $\sqrt{8{,}3} \approx 2{,}9$ cm
b $83 \text{ mm}^2 = 0{,}83 \text{ cm}^2$
lengte zijde van vierkant $KLMN$ is $\sqrt{0{,}83} \approx 0{,}91$ cm
c lengte zijde van vierkant $PQRS$ is $\sqrt{600}$ dm
omtrek $= 4 \cdot \sqrt{600} \approx 98{,}0$ dm

8
a $3^2 + 4^2 = 9 + 16 = 25$ en $5^2 = 25$, dus $3^2 + 4^2 = 5^2$ is juist.
b $4^2 + 5^2 = 16 + 25 = 41$ en $6^2 = 36$, dus $4^2 + 5^2 = 6^2$ is niet juist.
c $10^2 - 8^2 = 100 - 64 = 36$ en $6^2 = 36$, dus $10^2 - 8^2 = 6^2$ is juist.
d $13^2 - 12^2 = 169 - 144 = 25$ en $5^2 = 25$, dus $13^2 - 12^2 = 5^2$ is juist.
e $17^2 - 15^2 = 289 - 225 = 64$ en $8^2 = 64$, dus $17^2 - 15^2 = 8^2$ is juist.
f $15^2 + 20^2 = 225 + 400 = 625$ en $25^2 = 625$, dus $15^2 + 20^2 = 25^2$ is juist.
g $12^2 + 14^2 = 144 + 196 = 340$ en $16^2 = 256$, dus $12^2 + 14^2 = 16^2$ is niet juist.
h $3^2 + 2^2 = 9 + 4 = 13$ en $5^2 = 25$, dus $3^2 + 2^2 = 5^2$ is niet juist.
i $12^2 + 16^2 = 144 + 256 = 400$ en $20^2 = 400$, dus $12^2 + 16^2 = 20^2$ is juist.

9
a $x + 5^2 = 6^2$
$x + 25 = 36$
$-25 \quad -25$
$x = 11$
b $x = 8^2 - 2^2$
$x = 64 - 4 = 60$
c $x = 7^2 + 4^2$
$x = 49 + 16 = 65$
d $6^2 + x = 9^2$
$36 + x = 81$
$-36 \quad -36$
$x = 45$

10 **a** $\triangle ABD$ met $\angle D = 90°$ en $\triangle ACD$ met $\angle D = 90°$
b $\triangle PQR$ met $\angle R = 90°$
$\triangle QST$ met $\angle T = 90°$
$\triangle RST$ met $\angle T = 90°$
$\triangle PRS$ met $\angle S = 90°$
$\triangle QRS$ met $\angle S = 90°$

5.1 Rechthoekige driehoeken

Bladzijde 10

1 **a** aantal rode vierkanten = 16
aantal gele vierkanten = 9
aantal blauwe vierkanten = 25
b $16 + 9 = 25$
Dat de uitkomst van de som van het rode en gele vierkant precies gelijk is aan het aantal vierkantjes van het blauwe vierkant.

2 **a** oppervlakte groene vierkant = $90 + 35 = 125$ mm^2
b oppervlakte groene vierkant = $50 + 17 = 67$ mm^2
c oppervlakte groene vierkant = $140 - 120 = 20$ mm^2
d oppervlakte groene vierkant = $72 - 30 = 42$ mm^2

Bladzijde 11

3 **a** opp vierkant I = $1{,}6^2 = 2{,}56$ cm^2
opp vierkant II = $2{,}4^2 = 5{,}76$ cm^2
opp vierkant III = opp vierkant I + opp vierkant II = $2{,}56 + 5{,}76 = 8{,}32$ cm^2
b oppervlakte vierkant III = $8{,}32$ cm^2, dus $BC^2 = 8{,}32$, dus zijde $BC = \sqrt{8{,}32} \approx 2{,}9$ cm

Bladzijde 12

4 **a** $GI^2 + HI^2 = GH^2$
b $BC^2 + BD^2 = CD^2$

5 $\triangle ABC$: $AB^2 + BC^2 = AC^2$
$\triangle ABD$: $AB^2 + AD^2 = BD^2$

5.2 Zijden van rechthoekige driehoeken berekenen

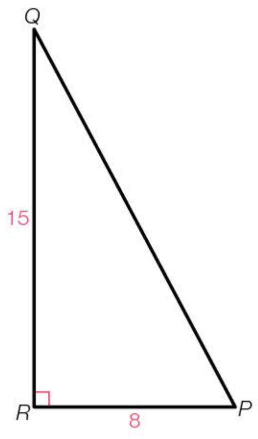
Bladzijde 13

6 **a** Uit $KL^2 = 10$ volgt $KL = \sqrt{10} \approx 3{,}16$.
b Uit $PQ^2 = 49$ volgt $PQ = \sqrt{49} = 7$.
c Uit $DE^2 = 8{,}4$ volgt $DE = \sqrt{8{,}4} \approx 2{,}90$.

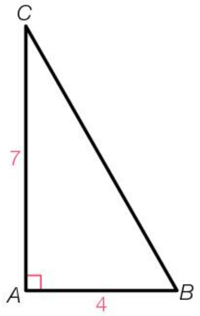
7 **a** $AB^2 + AC^2 = BC^2$
b $4^2 + 3^2 = BC^2$
$BC^2 = 16 + 9 = 25$
c $BC = \sqrt{25} = 5$

Bladzijde 14

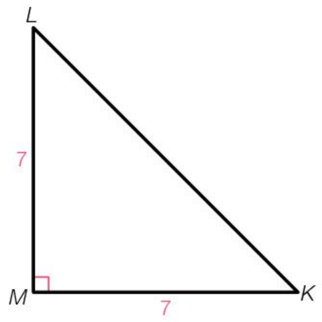
8 **a** Zie de schets hiernaast.
$\angle R = 90°$, dus $PR^2 + QR^2 = PQ^2$
$8^2 + 15^2 = PQ^2$
$64 + 225 = PQ^2$
$PQ^2 = 289$
$PQ = \sqrt{289} = 17$ cm



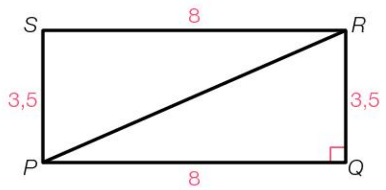
b Zie de schets hiernaast.
$\angle A = 90°$, dus $AB^2 + AC^2 = BC^2$
$4^2 + 7^2 = BC^2$
$16 + 49 = BC^2$
$BC^2 = 65$
$BC = \sqrt{65} \approx 8{,}1$ cm



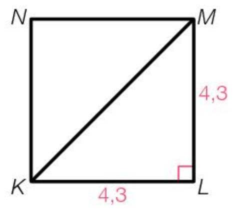
c Zie de schets hiernaast.
$\angle M = 90°$, dus $KM^2 + LM^2 = KL^2$
$7^2 + 7^2 = KL^2$
$49 + 49 = KL^2$
$KL^2 = 98$
$KL = \sqrt{98} \approx 9{,}9$ cm



9 **a** Zie de schets hiernaast.
$\angle Q = 90°$, dus $PQ^2 + QR^2 = PR^2$
$8^2 + 3{,}5^2 = PR^2$
$64 + 12{,}25 = PR^2$
$PR^2 = 76{,}25$
$PR = \sqrt{76{,}25} \approx 8{,}73$ cm



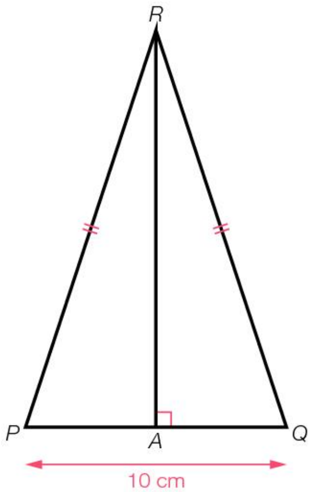
b Zie de schets hiernaast.
$\angle L = 90°$, dus $KL^2 + LM^2 = KM^2$
$4{,}3^2 + 4{,}3^2 = KM^2$
$18{,}49 + 18{,}49 = KM^2$
$KM^2 = 36{,}98$
$KM = \sqrt{36{,}98} \approx 6{,}08$ cm



10 In $\triangle ASD$ is $\angle S = 90°$, dus $AS^2 + SD^2 = AD^2$
$9^2 + 5^2 = AD^2$
$81 + 25 = AD^2$
$AD^2 = 106$
$AD = \sqrt{106} = 10{,}29...$ cm
$AB = AD = 10{,}29...$ cm (vlieger)
In $\triangle CSD$ is $\angle S = 90°$, dus $CS^2 + DS^2 = CD^2$
$4^2 + 5^2 = CD^2$
$16 + 25 = CD^2$
$CD^2 = 41$
$CD = \sqrt{41} = 6{,}40...$ cm
$BC = CD = 6{,}40...$ cm (vlieger)
omtrek $= AD + AB + CD + BC = 2 \cdot 10{,}29... + 2 \cdot 6{,}40... \approx 33{,}4$ cm

11 *Aanpak*
- Maak een schets met daarin $\triangle PQR$.
- Verwerk de gegevens in de figuur.

Uitwerking

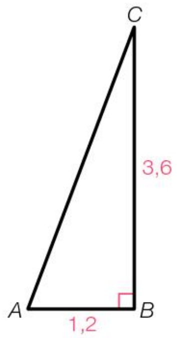


opp $\triangle PQR = \frac{1}{2} \cdot$ zijde $\cdot$ bijbehorende hoogte $= \frac{1}{2} \cdot PQ \cdot AR$

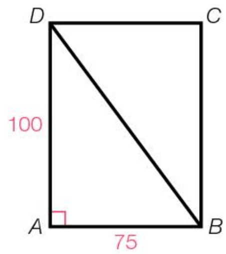
$60 = \frac{1}{2} \cdot 10 \cdot AR$

$60 = 5 \cdot AR$
$AR = 60 : 5 = 12$ cm
$AP = AQ = 10 : 2 = 5$ cm
$\angle A = 90°$, dus $AP^2 + AR^2 = PR^2$
$5^2 + 12^2 = AB^2$
$25 + 144 = AB^2$
$PR^2 = 169$
$PR = \sqrt{169} = 13$ cm
$QR = PR = 13$ cm
omtrek $\triangle PQR = 10 + 13 + 13 = 36$ cm

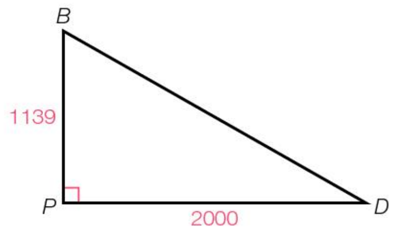
12 Zie de schets hiernaast.
$\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$
$1{,}2^2 + 3{,}6^2 = AC^2$
$1{,}44 + 12{,}96 = AC^2$
$AC^2 = 14{,}4$
$AC = \sqrt{14{,}4} = 3{,}794...$ m
De ladder steekt $4{,}5 - 3{,}794... \approx 0{,}71$ m $= 71$ cm boven de schutting uit.



13 Zie de schets hiernaast.
$\angle A = 90°$, dus $AB^2 + AD^2 = BD^2$
$75^2 + 100^2 = BD^2$
$5625 + 10\,000 = BD^2$
$BD^2 = 15\,625$
$BD = \sqrt{15625} \approx 125$ cm
De diagonaal van het raam is 125 cm.
Dus het tafelblad met een breedte van 122 cm past naar binnen.



14 Het hoogteverschil tussen berg- en dalstation is $1850 - 711 = 1139$ m.
Zie de schets hiernaast.



$\angle P = 90°$, dus $PD^2 + PB^2 = BD^2$

$2000^2 + 1139^2 = BD^2$

$4\,000\,000 + 1\,297\,321 = BD^2$

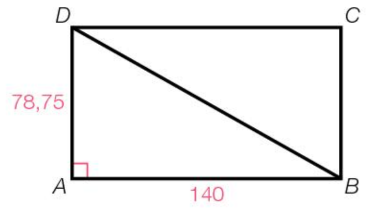
$BD^2 = 5\,297\,321$

$BD = \sqrt{5\,297\,321} \approx 2302$ m

De lengte van de gondelbaan is 2302 meter.

Bladzijde 15

15 De breedte is 140 cm en de verhouding breedte : hoogte = 16 : 9,
dus de hoogte is $\frac{9}{16} \cdot 140 = 78{,}75$ cm. Zie de schets hiernaast.



$\angle A = 90°$, dus $AB^2 + AD^2 = BD^2$

$140^2 + 78{,}75^2 = BD^2$

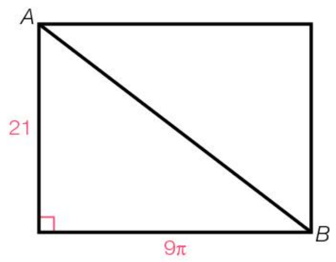
$19\,600 + 6201{,}5625 = BD^2$

$BD^2 = 25\,801{,}5625$

$BD = \sqrt{25\,801{,}5625} = 160{,}62...$ cm

De ideale kijkafstand is $3 \cdot 160{,}62... \approx 482$ cm.

16 omtrek cirkel = $\pi \cdot$ diameter = $\pi \cdot 9 = 9\pi$
Hiernaast zie je een schets van een uitslag van de cilindermantel.



Er geldt $(9\pi)^2 + 21^2 = AB^2$

$81\pi^2 + 441 = AB^2$

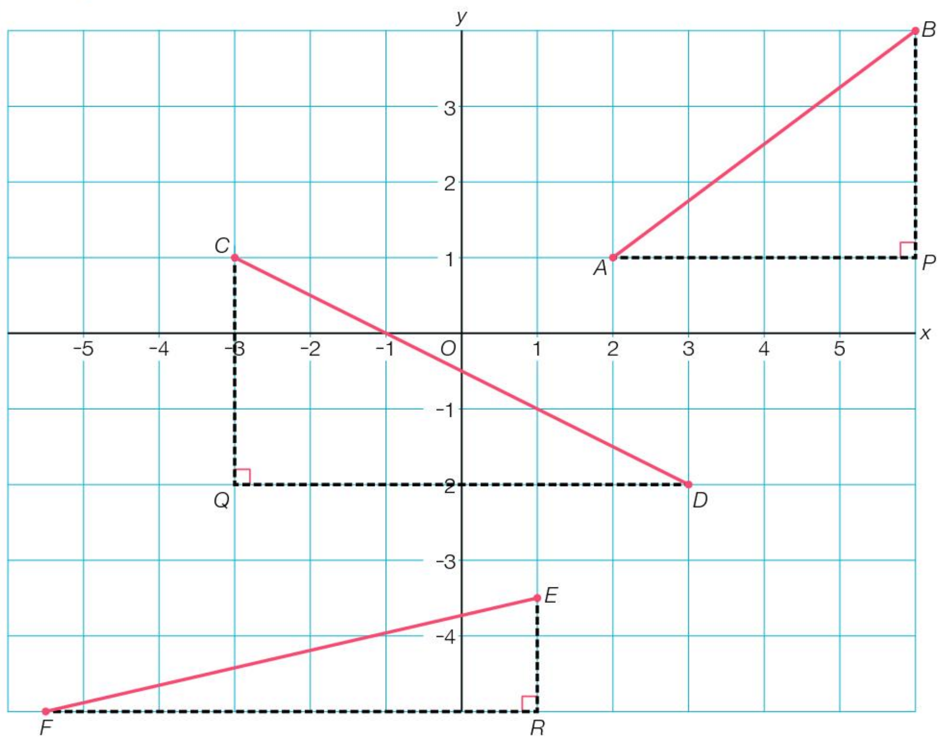
$AB^2 = 1240{,}43...$

$AB = \sqrt{1240{,}43...} \approx 35{,}2$ cm

De lengte van het touwtje is 352 mm.

Bladzijde 16

17



a $\angle P = 90°$, dus $AP^2 + BP^2 = AB^2$

$4^2 + 3^2 = AB^2$

$16 + 9 = AB^2$

$AB^2 = 25$

$AB = \sqrt{25} = 5$

b $\angle Q = 90°$, dus $CQ^2 + DQ^2 = CD^2$
$3^2 + 6^2 = CD^2$
$9 + 36 = CD^2$
$CD^2 = 45$
$CD = \sqrt{45} \approx 6{,}71$

c $\angle R = 90°$, dus $ER^2 + FR^2 = EF^2$
$1{,}5^2 + 6{,}5^2 = EF^2$
$2{,}25 + 42{,}25 = EF^2$
$EF^2 = 44{,}5$
$EF = \sqrt{44{,}5} \approx 6{,}67$

18 AB: $5^2 + 2^2 = AB^2$
$25 + 4 = AB^2$
$AB^2 = 29$
$AB = \sqrt{29} = 5{,}385...$
BC: $4^2 + 3^2 = BC^2$
$16 + 9 = BC^2$
$BC^2 = 25$
$BC = \sqrt{25} = 5$
CD: $2^2 + 2^2 = CD^2$
$4 + 4 = CD^2$
$CD^2 = 8$
$CD = \sqrt{8} = 2{,}828...$
AD: $3^2 + 7^2 = AD^2$
$9 + 49 = AD^2$
$AD^2 = 58$
$AD = \sqrt{58} = 7{,}615...$
De omtrek van vierhoek $ABCD = 5{,}385... + 5 + 2{,}828... + 7{,}615... \approx 20{,}8$

19 $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$
$5^2 + BC^2 = 6^2$
$25 + BC^2 = 36$
$-25 \qquad -25$
$BC^2 = 36 - 25$
Dus de beweringen b en e zijn juist.

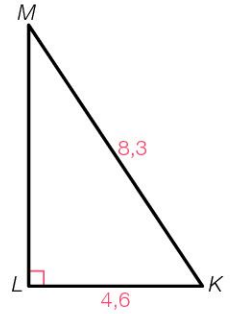
Bladzijde 17

20 $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$
$5^2 + BC^2 = 6^2$
$25 + BC^2 = 36$
$-25 \qquad -25$
$BC^2 = 11$
$BC = \sqrt{11} \approx 3{,}3$ cm

$\angle E = 90°$, dus $DE^2 + EF^2 = DF^2$
$DE^2 + 1^2 = 4^2$
$DE^2 + 1 = 16$
$-1 \quad -1$
$DE^2 = 15$
$DE = \sqrt{15} \approx 3{,}9$ cm

$\angle P = 90°$, dus $PQ^2 + QR^2 = PR^2$
$2^2 + QR^2 = 3^2$
$4 + QR^2 = 9$
$-4 \qquad -4$
$QR^2 = 5$
$QR = \sqrt{5} \approx 2{,}2$ cm

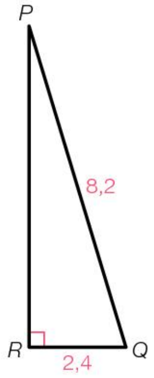
21 **a** Zie de schets hiernaast.



$\angle L = 90°$, dus $KL^2 + LM^2 = KM^2$

$$4{,}6^2 + LM^2 = 8{,}3^2$$
$$21{,}16 + LM^2 = 68{,}89$$
$$-\ 21{,}16 \qquad -\ 21{,}16$$
$$LM^2 = 47{,}73$$
$$LM = \sqrt{47{,}73} \approx 6{,}9 \text{ cm}$$

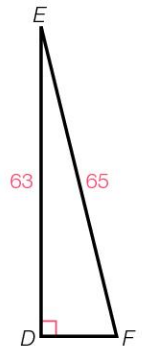
b Zie de schets hiernaast.



$\angle R = 90°$, dus $PR^2 + QR^2 = PQ^2$

$$PR^2 + 2{,}4^2 = 8{,}2^2$$
$$PR^2 + 5{,}76 = 67{,}24$$
$$-\ 5{,}76 \quad -\ 5{,}76$$
$$PR^2 = 61{,}48$$
$$PR = \sqrt{61{,}48} \approx 7{,}8 \text{ cm}$$

c Zie de schets hiernaast.



$\angle D = 90°$, dus $DE^2 + DF^2 = EF^2$

$$63^2 + DF^2 = 65^2$$
$$3969 + DF^2 = 4225$$
$$-\ 3969 \qquad -\ 3969$$
$$DF^2 = 256$$
$$DF = \sqrt{256} = 16 \text{ cm}$$

22 $\angle C = 90°$, dus $CD^2 + BC^2 = BD^2$

$$CD^2 + (2600 - 1750)^2 = 3300^2$$
$$CD^2 + 722\,500 = 10\,890\,000$$
$$-\ 722\,500 \quad -\ 722\,500$$
$$CD^2 = 10\,167\,500$$
$$CD = \sqrt{10\,167\,500} = 3188{,}6... \text{ m}$$

Dus de horizontale afstand CD afgerond op tientallen is 3190 m.

23 In $\triangle AED$ is $\angle E = 90°$, dus $AE^2 + DE^2 = AD^2$

$$AE^2 + 4{,}8^2 = 6^2$$
$$AE^2 + 23{,}04 = 36$$
$$-\ 23{,}04 \quad -\ 23{,}04$$
$$AE^2 = 12{,}96$$
$$AE = \sqrt{12{,}96} = 3{,}6 \text{ cm}$$

In $\triangle ABF$ is $\angle F = 90°$, dus $AF^2 + BF^2 = AB^2$

$$AF^2 + 4{,}8^2 = 8^2$$
$$AF^2 + 23{,}04 = 64$$
$$-\ 23{,}04 \quad -\ 23{,}04$$
$$AF^2 = 40{,}96$$
$$AF = \sqrt{40{,}96} = 6{,}4 \text{ cm}$$

$EF = AF - AE = 6{,}4 - 3{,}6 = 2{,}8$ cm

Bladzijde 18

24 **a** $64 + 35 = 99$

$99 \neq 100$, dus de rode driehoek is niet rechthoekig.

b $105 + 39 = 144$

De oppervlakte van het grote vierkant is ook 144, dus de rode driehoek is rechthoekig.

c $78 + 78 = 156$

$156 \neq 158$, dus de rode driehoek is niet rechthoekig.

Bladzijde 19

25 **a** $\left.\begin{array}{l} AC^2 + BC^2 = 30^2 + 1{,}5^2 = 1056{,}25 \\ AB^2 = 33{,}75^2 = 1139{,}0625 \end{array}\right\} AC^2 + BC^2 \neq AB^2$, dus $\triangle ABC$ is niet rechthoekig.

b $\left.\begin{array}{l} KM^2 + LM^2 = 12^2 + 15^2 = 369 \\ KL^2 = 19^2 = 361 \end{array}\right\} KM^2 + LM^2 \neq KL^2$, dus $\triangle KLM$ is niet rechthoekig.

c $\left.\begin{array}{l} PQ^2 + PR^2 = 35^2 + 12^2 = 1369 \\ QR^2 = 37^2 = 1369 \end{array}\right\} PQ^2 + PR^2 = KL^2$, dus $\angle P = 90°$.

26 In $\triangle KMP$ is $\angle M = 90°$, dus $KM^2 + MP^2 = KP^2$

$$\begin{array}{l} KM^2 + 18^2 = 30^2 \\ KM^2 + 324 = 900 \\ \quad -324 \quad -324 \\ KM^2 = 576 \end{array}$$

In $\triangle KLQ$ is $\angle L = 90°$, dus $KL^2 + LQ^2 = KQ^2$

$$\begin{array}{l} KL^2 + 7{,}5^2 = 12{,}5^2 \\ KL^2 + 56{,}25 = 156{,}25 \\ \quad -56{,}25 \quad -56{,}25 \\ KL^2 = 100 \end{array}$$

In $\triangle LMR$ is $\angle R = 90°$, dus $LR^2 + MR^2 = LM^2$

$$\begin{array}{l} 15{,}6^2 + 20{,}8^2 = LM^2 \\ 243{,}36 + 432{,}64 = LM^2 \\ LM^2 = 676 \end{array}$$

$\left.\begin{array}{l} KL^2 + KM^2 = 100 + 576 = 676 \\ LM^2 = 676 \end{array}\right\} KL^2 + KM^2 = LM^2$

Dus $\triangle KLM$ is rechthoekig.

27 In $\triangle ACD$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$

$$\begin{array}{l} AD^2 + 12^2 = 13^2 \\ AD^2 + 144 = 169 \\ \quad -144 \quad -144 \\ AD^2 = 25 \\ AD = \sqrt{25} = 5 \text{ mm} \end{array}$$

In $\triangle BCD$ is $\angle D = 90°$, dus $BD^2 + CD^2 = BC^2$

$$\begin{array}{l} 35^2 + 12^2 = BC^2 \\ 1225 + 144 = BC^2 \\ BC^2 = 1369 \\ BC = \sqrt{1369} = 37 \text{ mm} \end{array}$$

$AB = 5 + 35 = 40$ mm.

$\left.\begin{array}{l} AC^2 + BC^2 = 13^2 + 37^2 = 1538 \\ AB^2 = 40^2 = 1600 \end{array}\right\} AC^2 + BC^2 \neq AB^2$, dus $\triangle ABC$ is niet rechthoekig.

28 $\left.\begin{array}{l} \text{In } \triangle CDE \text{ is } DE^2 + CE^2 = (2\sqrt{3})^2 + 6^2 = 12 + 36 = 48 \\ CD^2 = (4\sqrt{3})^2 = 48 \end{array}\right\} DE^2 + CE^2 = CD^2$, dus $\angle E_2 = 90°$.

In $\triangle BED$ is $\angle E_1 = 90°$, dus $BE^2 + DE^2 = BD^2$

$$\begin{array}{l} (2\sqrt{3})^2 + 2^2 = BD^2 \\ 12 + 4 = BD^2 \\ BD^2 = 16 \\ BD = \sqrt{16} = 4 \end{array}$$

$\angle D_3 = 180° - 90° - 30° = 60°$ (hoekensom driehoek)

$\angle D_1 = 180° - 60° - 30° = 90°$ (gestrekte hoek)

In $\triangle ABD$ is $\angle D_1 = 90°$, dus $AD^2 + BD^2 = AB^2$

$$\begin{array}{l} AD^2 + 4^2 = (4\sqrt{2})^2 \\ AD^2 + 16 = 32 \\ \quad -16 \quad -16 \\ AD^2 = 16 \\ AD = \sqrt{16} = 4 \end{array}$$

$AD = BD = 4$, dus $\triangle ABD$ is gelijkbenig met $\angle D_1 = 90°$.
$\angle A + \angle B_1 = 180° - 90° = 90°$ (hoekensom driehoek)
$\angle A = \angle B_1 = 90° : 2 = 45°$ (basishoeken)

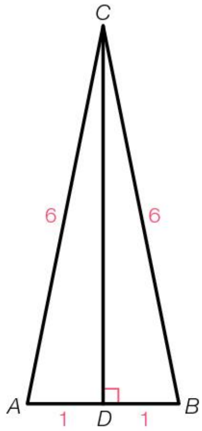
5.3 De stelling van Pythagoras toepassen

Bladzijde 20

29 De beweringen c en d zijn waar.

Bladzijde 21

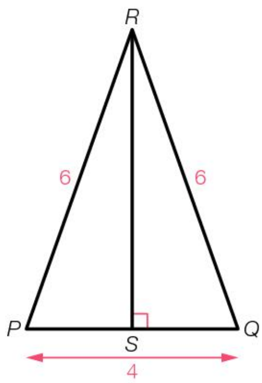
30 Zie de schets hiernaast.
$AD = 2 : 2 = 1$ m
In $\triangle ACD$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$
$1^2 + CD^2 = 6^2$
$1 + CD^2 = 36$
$-1 \qquad -1$
$CD^2 = 35$
$CD = \sqrt{35} = 5{,}91...$ m
De hoogte van de aula is $5{,}91... + 1 \approx 6{,}9$ meter.



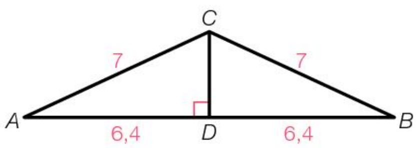
31 **a** $AD = 12 : 2 = 6$ cm
In $\triangle ACD$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$
$6^2 + CD^2 = 10^2$
$36 + CD^2 = 100$
$-36 \qquad -36$
$CD^2 = 64$
$CD = \sqrt{64} = 8$ cm

b opp $\triangle ABC = \frac{1}{2} \cdot 12 \cdot 8 = 48$ cm^2

32 Zie de schets hiernaast.
$PS = 4 : 2 = 2$ cm
In $\triangle PSR$ is $\angle S = 90°$, dus $PS^2 + RS^2 = PR^2$
$2^2 + RS^2 = 6^2$
$4 + RS^2 = 36$
$-4 \qquad -4$
$RS^2 = 32$
$RS = \sqrt{32} \approx 5{,}7$ cm

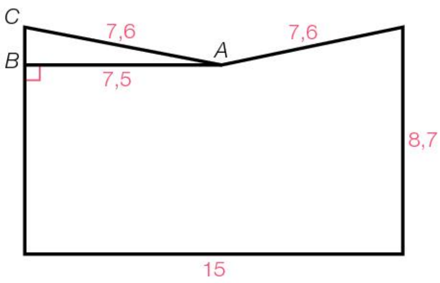


33 Zie de schets hiernaast.
$AD = 12{,}8 : 2 = 6{,}4$ m
In $\triangle ACD$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$
$6{,}4^2 + CD^2 = 7^2$
$40{,}96 + CD^2 = 49$
$-40{,}96 \qquad -40{,}96$
$CD^2 = 8{,}04$
$CD = \sqrt{8{,}04} = 2{,}835...$ m
De hoogte van de kas is $3{,}2 + 2{,}835... \approx 6{,}04$ meter ofwel 604 cm.



Bladzijde 22

34 Zie de schets hiernaast.

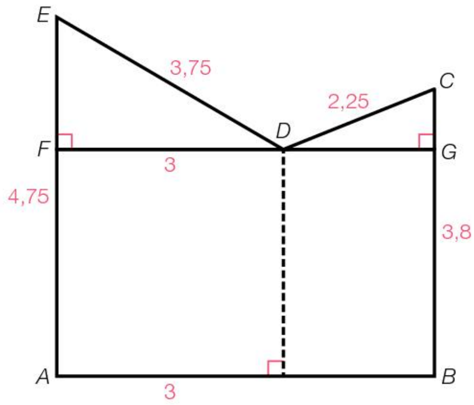


$AB = 15 : 2 = 7{,}5$ m
$AC = 15{,}2 : 2 = 7{,}6$ m
In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

$$
\begin{aligned}
7{,}5^2 + BC^2 &= 7{,}6^2\\
56{,}25 + BC^2 &= 57{,}76\\
\mathbf{-\,56{,}25} \qquad & \quad \mathbf{-\,56{,}25}\\
BC^2 &= 1{,}51\\
BC &= \sqrt{1{,}51} = 1{,}228...\text{ m}
\end{aligned}
$$

$8{,}7 - 1{,}228... - 0{,}9 \approx 6{,}57$ m
Dus een 6,5 meter hoge takelwagen kan onder het verkeerslicht door rijden.

35 Zie de schets hiernaast.



In $\triangle DEF$ is $\angle F = 90°$, dus $DF^2 + EF^2 = DE^2$

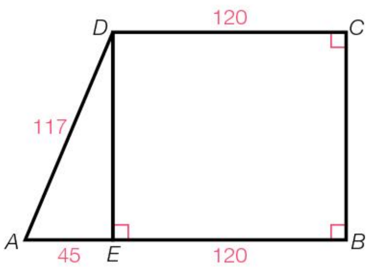
$$
\begin{aligned}
3^2 + EF^2 &= 3{,}75^2\\
9 + EF^2 &= 14{,}0625\\
\mathbf{-\,9} \qquad & \quad \mathbf{-\,9}\\
EF^2 &= 5{,}0625\\
EF &= \sqrt{5{,}0625} = 2{,}25\text{ m}
\end{aligned}
$$

$AF = 4{,}75 - 2{,}25 = 2{,}5$ m.
$CG = 3{,}8 - 2{,}5 = 1{,}3$ m.
In $\triangle DGC$ is $\angle G = 90°$, dus $DG^2 + CG^2 = CD^2$

$$
\begin{aligned}
DG^2 + 1{,}3^2 &= 2{,}25^2\\
DG^2 + 1{,}69 &= 5{,}0625\\
\mathbf{-\,1{,}69} \qquad & \quad \mathbf{-\,1{,}69}\\
DG^2 &= 3{,}3725\\
DG &= \sqrt{3{,}3725} = 1{,}836...\text{ m}
\end{aligned}
$$

De breedte van de steeg is $3 + 1{,}836... \approx 4{,}84$ m.

36 **a** Zie de schets hiernaast.



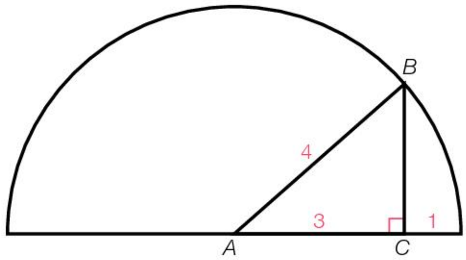
$AE = 165 - 120 = 45$ m
In $\triangle ADE$ is $\angle E = 90°$, dus $AE^2 + DE^2 = AD^2$

$$
\begin{aligned}
45^2 + DE^2 &= 117^2\\
2025 + DE^2 &= 13\,689\\
\mathbf{-\,2025} \qquad & \quad \mathbf{-\,2025}\\
DE^2 &= 11\,664\\
DE &= \sqrt{11\,664} = 108\text{ m}
\end{aligned}
$$

De ontbrekende afstand is 108 meter.

b Het parcours is $165 + 108 + 120 + 117 = 510$ meter.
Mirjam loopt $8 \cdot 510 = 4080$ meter.
Daar doet ze $27 \cdot 60 + 40 = 1660$ seconden over.
Ze loopt dus $\frac{4080}{1660} = 2{,}45...$ m/sec.
1 m/sec = 3600 m/uur = 3,6 km/uur
Haar gemiddelde snelheid is $2{,}45... \cdot 3{,}6 \approx 8{,}8$ km per uur.

37 Zie de schets hiernaast met A het middelpunt van de halve cirkel en AB een straal.



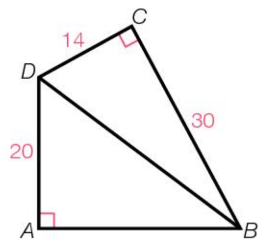
In $\triangle ABC$ is $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$

$$
\begin{aligned}
3^2 + BC^2 &= 4^2\\
9 + BC^2 &= 16\\
\mathbf{-\,9} \qquad & \quad \mathbf{-\,9}\\
BC^2 &= 7\\
BC &= \sqrt{7} \approx 2{,}6\text{ m}
\end{aligned}
$$

De maximale hoogte van de camper is 2,6 m.

Bladzijde 23

38 **a** Zie de schets hiernaast.



In $\triangle BCD$ is $\angle C = 90°$, dus $BC^2 + CD^2 = BD^2$

$30^2 + 14^2 = BD^2$

$900 + 196 = BD^2$

$BD^2 = 1096$

In $\triangle ABD$ is $\angle A = 90°$, dus $AB^2 + AD^2 = BD^2$

$AB^2 + 20^2 = 1096$

$AB^2 + 400 = 1096$

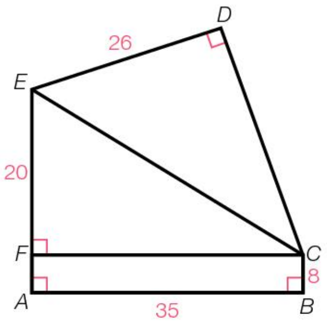
$-400 \quad -400$

$AB^2 = 696$

$AB = \sqrt{696} \approx 26$ m

De lengte van de onbekende zijde AB is 26 m.

b Zie de schets hiernaast.



In $\triangle CEF$ is $\angle F = 90°$, dus $CF^2 + EF^2 = CE^2$

$35^2 + 20^2 = CE^2$

$1225 + 400 = CE^2$

$CE^2 = 1625$

In $\triangle CDE$ is $\angle D = 90°$, dus $CD^2 + DE^2 = CE^2$

$CD^2 + 26^2 = 1625$

$CD^2 + 676 = 1625$

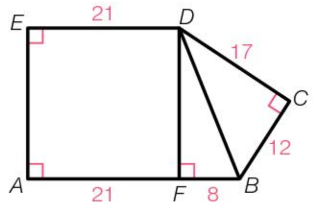
$-676 \quad -676$

$CD^2 = 949$

$CD = \sqrt{949} \approx 31$ m

De lengte van de onbekende zijde CD is 31 m.

c Zie de schets hiernaast.



In $\triangle BCD$ is $\angle C = 90°$, dus $BC^2 + CD^2 = BD^2$

$12^2 + 17^2 = BD^2$

$144 + 289 = BD^2$

$BD^2 = 433$

In $\triangle BDF$ is $\angle F = 90°$, dus $BF^2 + DF^2 = BD^2$

$8^2 + DF^2 = 433$

$64 + DF^2 = 433$

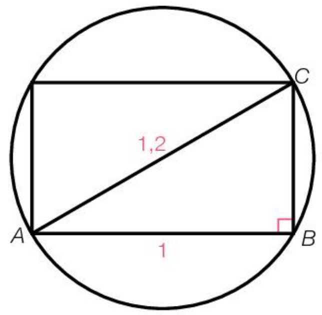
$-64 \quad -64$

$DF^2 = 369$

$DF = \sqrt{369} \approx 19$ m

De lengte van de onbekende zijde $AE = DF = 19$ m.

39 Zie de schets hiernaast.



In $\triangle ABC$ is $\angle C = 90°$, dus $AB^2 + BC^2 = AC^2$

$1^2 + BC^2 = 1{,}2^2$

$1 + BC^2 = 1{,}44$

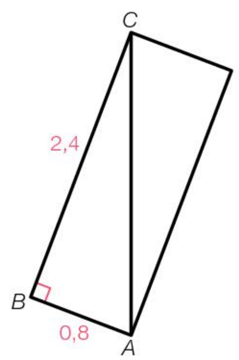
$-1 \quad -1$

$BC^2 = 0{,}44$

$BC = \sqrt{0{,}44} = 0{,}6633...$ m

De balk bestaat uit $1 \cdot 0{,}6633... \cdot 10 \approx 6{,}63$ m^3 hout.

40 Zie de schets hiernaast.



In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

$0{,}8^2 + 2{,}4^2 = AC^2$

$0{,}64 + 5{,}76 = AC^2$

$AC^2 = 6{,}4$

$AC = \sqrt{6{,}4} \approx 2{,}53$ m

Dit is meer dan 2,50 m, dus het is niet mogelijk de gekantelde kast rechtop te zetten.

5.4 Doorsneden

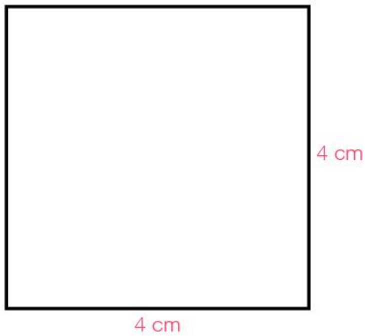
Bladzijde 24

41 Bij snede A hoort figuur 3.
Bij snede B hoort figuur 2.
Bij snede C hoort figuur 4.
Bij snede D hoort figuur 1.

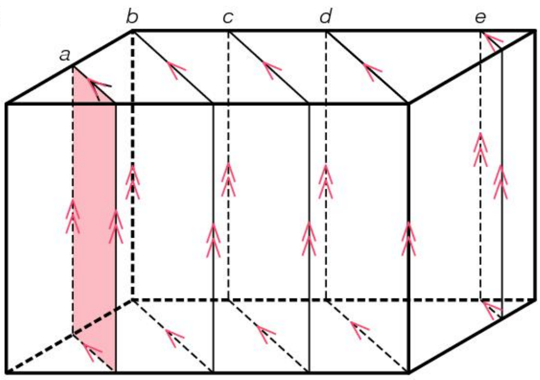
42 Bij lijn a hoort doorsnede 2.
Bij lijn b hoort doorsnede 3.
Bij lijn c hoort doorsnede 1.

Bladzijde 25

43 **a** Schaal 1 : 5. Je krijgt een vierkant met zijden van 4 cm.
b De doorsnede heeft de vorm van een rechthoek.
c Gijs heeft een hoekpunt van de kubus afgezaagd.



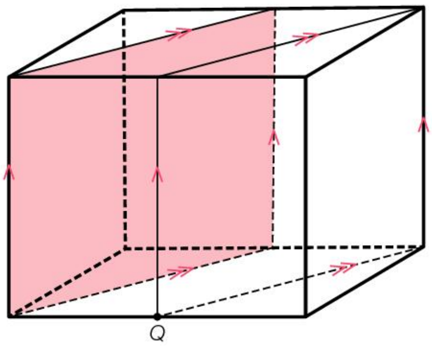
44 **a,b**



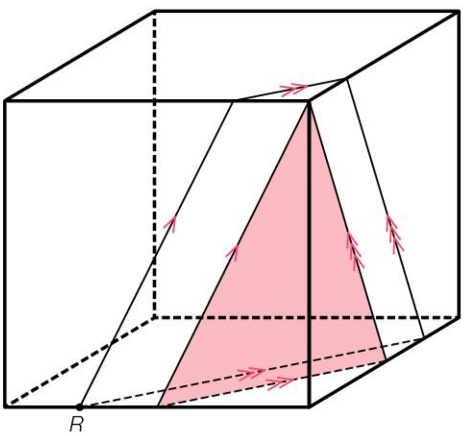
c De doorsneden bij b, c en d hebben dezelfde afmetingen.

Bladzijde 26

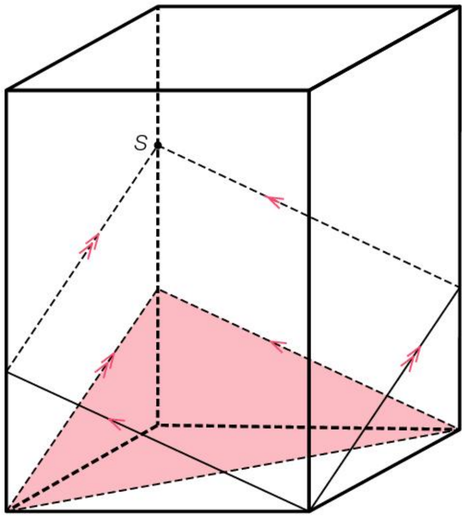
45 **a**



b

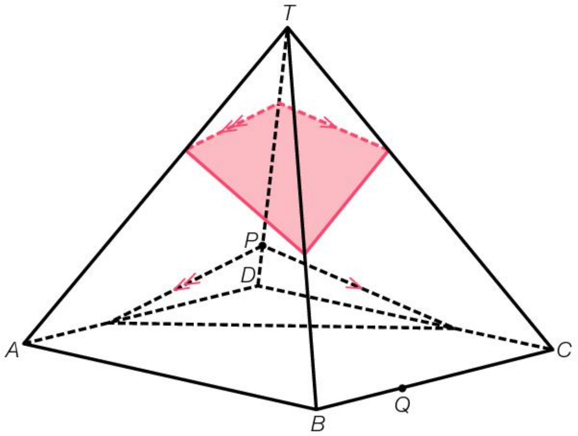


c

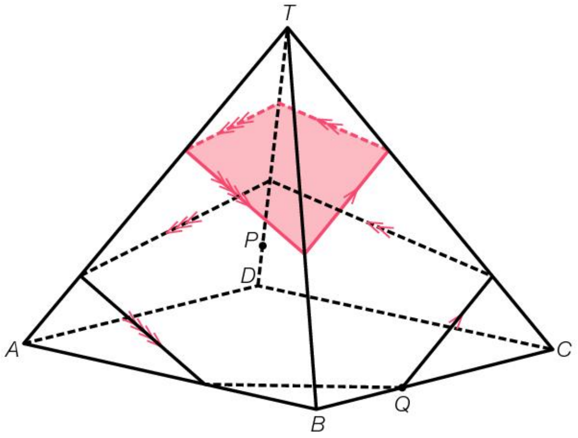


Bladzijde 27

46 **a**

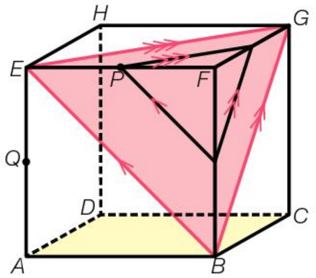


b

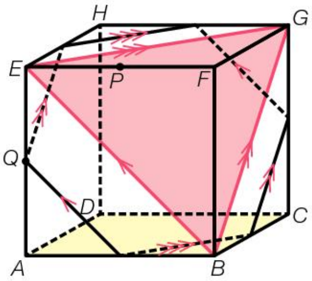


47 **a** 1, 2, 3, 4 en 5
b 1, 2, 3 en 5

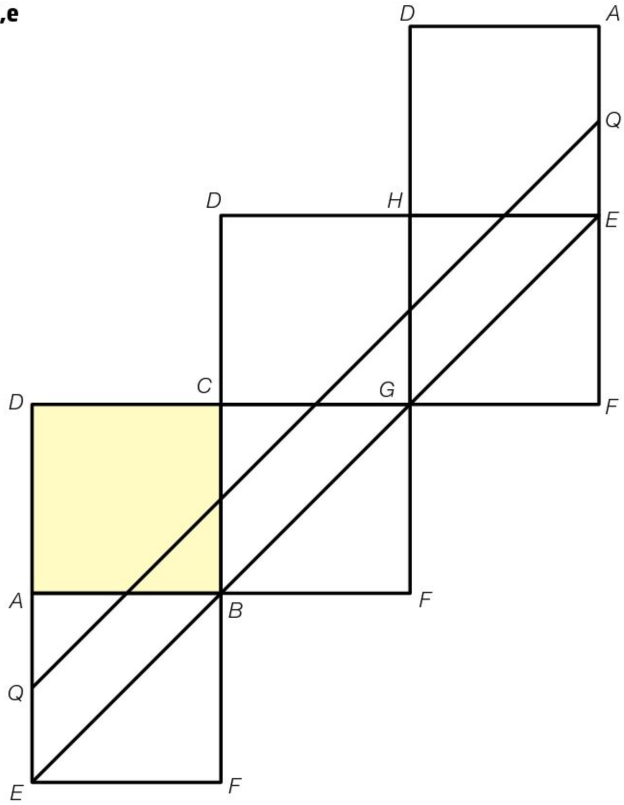
48 **a**



b



c,d,e



d In $\triangle BEF$ is $\angle F = 90°$, dus $BF^2 + EF^2 = BE^2$

$$6^2 + 6^2 = BE^2$$
$$36 + 36 = BE^2$$
$$BE^2 = 72$$
$$BE = \sqrt{72} = 8{,}48\ldots \text{ cm}$$

$BE = BG = EG$, dus de omtrek van $\triangle BGE$ is $3 \cdot 8{,}48\ldots \approx 25{,}5$ cm.

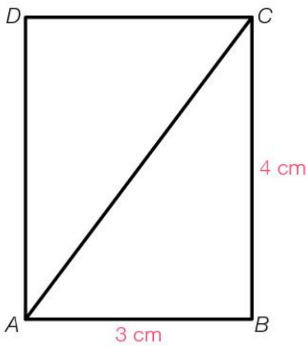
e In de uitslag zie je dat de omtrek van de doorsnede door Q gelijk is aan de omtrek van $\triangle BGE$.
Dus de omtrek van de doorsnede door Q is 25,5 cm.

5.5 Pythagoras in de ruimte

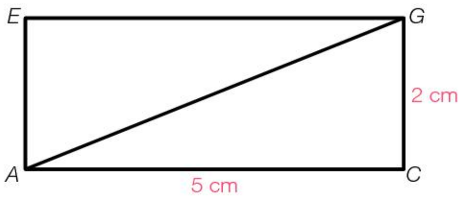
Bladzijde 28

49 **a** Zie de figuur hiernaast.
In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

$$3^2 + 4^2 = AC^2$$
$$9 + 16 = AC^2$$
$$AC^2 = 25$$
$$AC = \sqrt{25} = 5 \text{ cm}$$



b Vierhoek $ACGE$ is een rechthoek.
De zijden zijn 2 cm en 5 cm.
Zie de figuur hiernaast.



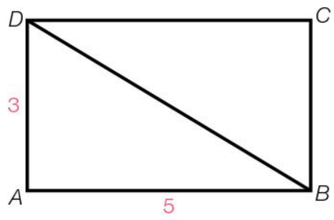
c In $\triangle ACG$ is $\angle C = 90°$, dus $AC^2 + CG^2 = AG^2$

$$5^2 + 2^2 = AG^2$$
$$25 + 4 = AG^2$$
$$AG^2 = 29$$
$$AG = \sqrt{29} \approx 5{,}39 \text{ cm}$$

Bladzijde 29

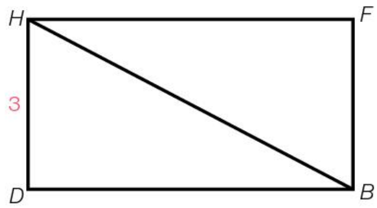
50 **a** Zie de figuur hiernaast.
In $\triangle ABD$ is $\angle A = 90°$, dus $AB^2 + AD^2 = BD^2$

$$5^2 + 3^2 = BD^2$$
$$25 + 9 = BD^2$$
$$BD^2 = 34$$



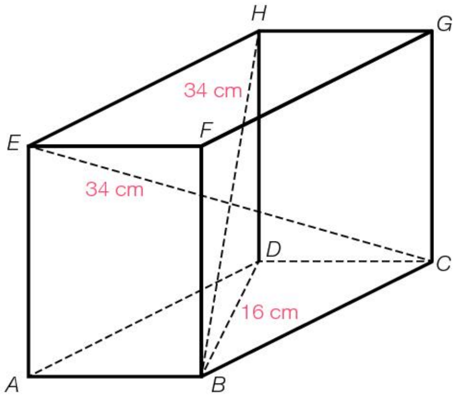
Zie de figuur hiernaast.
In $\triangle BDH$ is $\angle D = 90°$, dus $BD^2 + DH^2 = BH^2$

$$34 + 3^2 = BH^2$$
$$34 + 9 = BH^2$$
$$BH^2 = 43$$
$$BH = \sqrt{43} \approx 6{,}6 \text{ cm}$$



b De andere lichaamsdiagonalen zijn ook 6,6 cm.

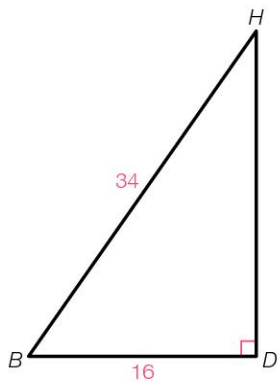
51 Schets balk $ABCD\ EFGH$, zie hiernaast.
$BH = CE = 34$ cm



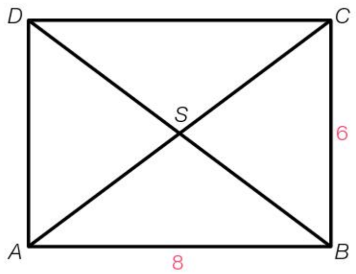
Schets $\triangle BDH$, zie hiernaast.
In $\triangle BDH$ is $\angle D = 90°$, dus $BD^2 + DH^2 = BH^2$

$$16^2 + DH^2 = 34^2$$
$$256 + DH^2 = 1156$$
$$-\,256 \qquad -\,256$$
$$DH^2 = 900$$
$$DH = \sqrt{900} = 30 \text{ cm}$$

De hoogte van de balk is 30 cm.



52 **a** Schets grondvlak, zie hiernaast.

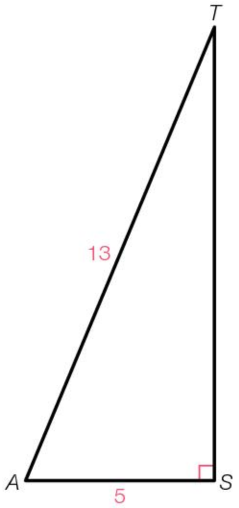


In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

$$8^2 + 6^2 = AC^2$$
$$64 + 36 = AC^2$$
$$AC^2 = 100$$
$$AC = \sqrt{100} = 10 \text{ cm}$$

AS is de helft van AC, dus $AS = 10 : 2 = 5$ cm.

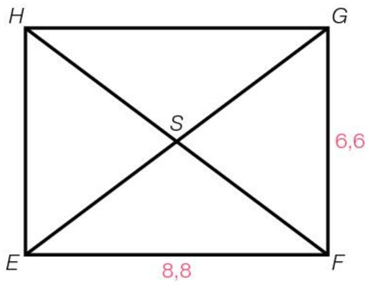
b Schets $\triangle AST$, zie hiernaast.



In $\triangle AST$ is $\angle S = 90°$, dus $AS^2 + ST^2 = AT^2$

$$5^2 + ST^2 = 13^2$$
$$25 + ST^2 = 169$$
$$-25 \qquad -25$$
$$ST^2 = 144$$
$$ST = \sqrt{144} = 12 \text{ cm}$$

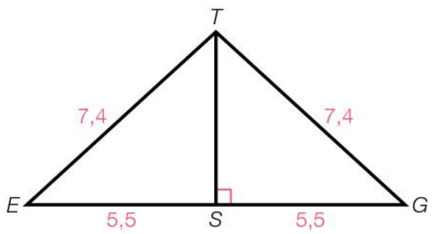
53 **a** Schets grondvlak piramide, zie hiernaast.



In $\triangle EFG$ is $\angle F = 90°$, dus $EF^2 + FG^2 = EG^2$

$$8{,}8^2 + 6{,}6^2 = EG^2$$
$$77{,}44 + 43{,}56 = EG^2$$
$$EG^2 = 121$$
$$EG = \sqrt{121} = 11 \text{ m}$$

Schets $\triangle EST$, zie hiernaast.



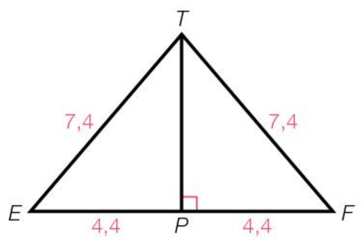
ES is de helft van EG, dus $ES = 11 : 2 = 5{,}5$ m.

In $\triangle EST$ is $\angle S = 90°$, dus $ES^2 + ST^2 = ET^2$

$$5{,}5^2 + ST^2 = 7{,}4^2$$
$$30{,}25 + ST^2 = 54{,}76$$
$$-30{,}25 \qquad -30{,}25$$
$$ST^2 = 24{,}51$$
$$ST = \sqrt{24{,}51} = 4{,}950... \text{ m}$$

Dus de hoogte van het huis is $3 + 4{,}950... \approx 7{,}95$ m.

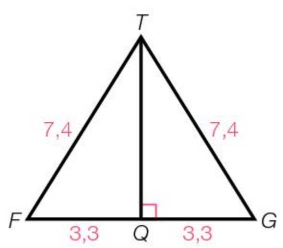
b Zie de schets hiernaast van $\triangle EFT$ met hulplijn PT.



In $\triangle EPT$ is $\angle P = 90°$, dus $EP^2 + PT^2 = ET^2$

$$4{,}4^2 + PT^2 = 7{,}4^2$$
$$19{,}36 + PT^2 = 54{,}76$$
$$-19{,}36 \qquad -19{,}36$$
$$PT^2 = 35{,}4$$
$$PT = \sqrt{35{,}4} = 5{,}94... \text{ m}$$

Zie de schets hiernaast van $\triangle FGT$ met hulplijn QT.



In $\triangle FQT$ is $\angle Q = 90°$, dus $FQ^2 + QT^2 = FT^2$

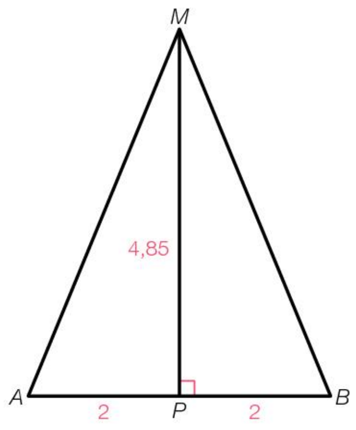
$$3{,}3^2 + QT^2 = 7{,}4^2$$
$$10{,}89 + QT^2 = 54{,}76$$
$$-10{,}89 \qquad -10{,}89$$
$$QT^2 = 43{,}87$$
$$QT = \sqrt{43{,}87} = 6{,}62... \text{ m}$$

totale oppervlakte dak $= 2 \cdot \text{opp } \triangle EFT + 2 \cdot \text{opp } \triangle FGT = 2 \cdot \frac{1}{2} \cdot EF \cdot PT + 2 \cdot \frac{1}{2} \cdot FG \cdot QT$

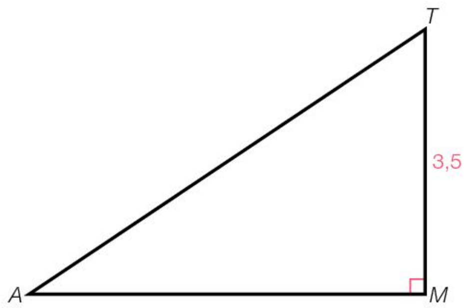
$$= 2 \cdot \frac{1}{2} \cdot 8{,}8 \cdot 5{,}94... + 2 \cdot \frac{1}{2} \cdot 6{,}6 \cdot 6{,}62... \approx 96{,}07 \text{ m}^2.$$

Bladzijde 30

54 **a** Het punt M is het snijpunt van AE en BF, dus M is het midden van de achthoek $ABCD\ EFGH$ en ligt loodrecht onder T.
$AB = 32 : 8 = 4$ m
Zie de schets hiernaast met hulplijn MP.
$AP = BP = 4 : 2 = 2$ m
$MP = 9{,}7 : 2 = 4{,}85$ m
In $\triangle APM$ is $\angle P = 90°$, dus $AP^2 + MP^2 = AM^2$
$2^2 + 4{,}85^2 = AM^2$
$4 + 23{,}5225 = AM^2$
$AM^2 = 27{,}5225$



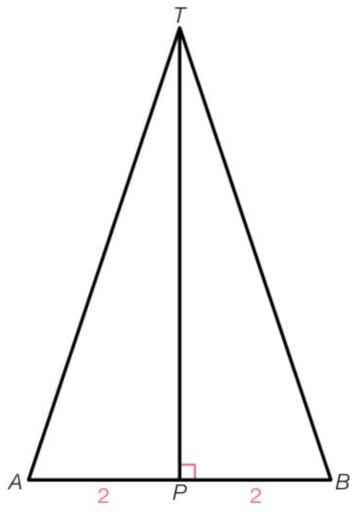
Zie de schets hiernaast.
$MT = 6 - 2{,}5 = 3{,}5$ m
In $\triangle AMT$ is $\angle M = 90°$, dus $AM^2 + MT^2 = AT^2$
$27{,}5225 + 3{,}5^2 = AT^2$
$27{,}5225 + 12{,}25 = AT^2$
$AT^2 = 39{,}7725$
$AT = \sqrt{39{,}7725} \approx 6{,}31$ m



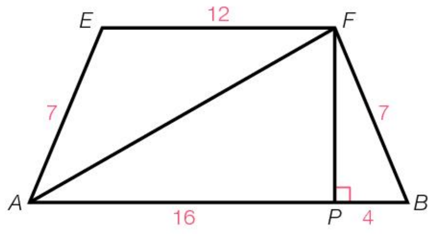
b Zie de schets hiernaast met hulplijn PT.
In $\triangle APT$ is $\angle P = 90°$, dus $AP^2 + PT^2 = AT^2$
$2^2 + PT^2 = 39{,}7725$
$4 + PT^2 = 39{,}7725$
$-4 \qquad -4$
$PT^2 = 35{,}7725$
$PT = \sqrt{35{,}7725} = 5{,}981...$

opp $\triangle ABT = \frac{1}{2} \cdot 4 \cdot 5{,}981... = 11{,}962...$

De oppervlakte van het dak is $8 \cdot 11{,}962... \approx 95{,}70$ m^2.



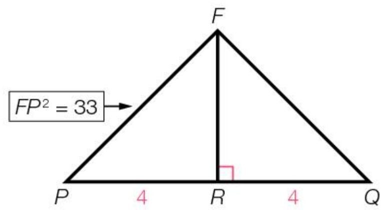
55 **a** Zie de schets hiernaast van het gelijkbenig trapezium $ABFE$ met hulplijn FP.
In $\triangle BFP$ is $\angle P = 90°$, dus $BP^2 + FP^2 = BF^2$
$4^2 + FP^2 = 7^2$
$16 + FP^2 = 49$
$-16 \qquad -16$
$FP^2 = 33$
In $\triangle AFP$ is $\angle P = 90°$, dus $AP^2 + FP^2 = AF^2$
$16^2 + 33 = AF^2$
$256 + 33 = AF^2$
$AF^2 = 289$
$AF = \sqrt{289} = 17$ m
De lengte van de kabel is 17 meter.



b Zie de schets hiernaast van de gelijkbenige driehoek FPQ met hulplijn FR, waarbij het punt R recht onder F ligt in het vlak $ABCD$.
In $\triangle FPR$ is $\angle R = 90°$, dus $PR^2 + FR^2 = FP^2$

$4^2 + FR^2 = 33$
$16 + FR^2 = 33$
$-16 \qquad -16$
$FR^2 = 17$
$FR = \sqrt{17} = 4{,}123...$ m

De afstand van de rand EF tot de grond is $6 + 4{,}123... \approx 10{,}12$ meter.

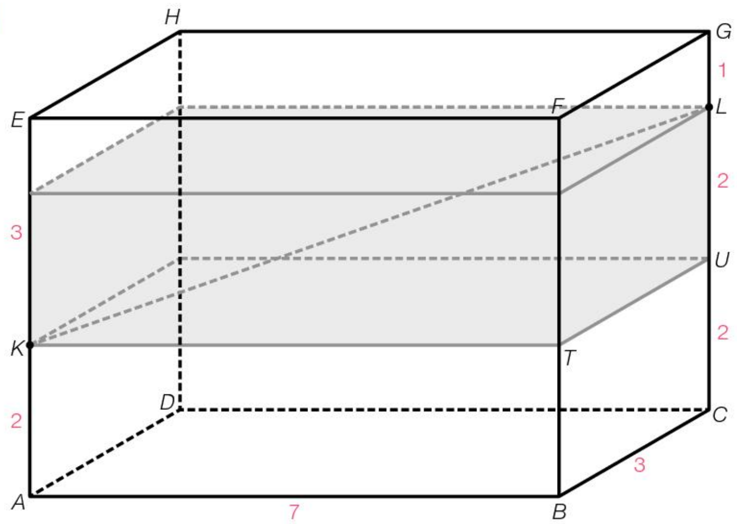


56 **a** $AB^2 + AD^2 = BD^2$
b $BD^2 + DH^2 = BH^2$
c In opgave a vind je dat $BD^2 = AB^2 + AD^2$ ofwel $BH^2 = AB^2 + AD^2 + DH^2$.
Vervang BD^2 in het antwoord van opgave b door $AB^2 + AD^2$.
Je krijgt $AB^2 + AD^2 + DH^2 = BH^2$.
$BH^2 = 6^2 + 2^2 + 3^2 = 36 + 4 + 9 = 49$
$BH = \sqrt{49} = 7$ cm

Bladzijde 32

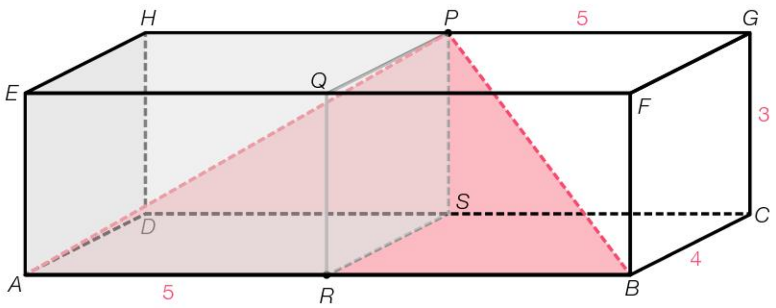
57 **a** $CE^2 = CB^2 + BA^2 + AE^2 = 4^2 + 7^2 + 5^2 = 90$
Dus $CE = \sqrt{90} \approx 9{,}5$ cm.
b Zie de schets hiernaast. $EP = 7 - 2 = 5$ cm
$DP^2 = DA^2 + AE^2 + EP^2 = 4^2 + 5^2 + 5^2 = 66$
Dus $DP = \sqrt{66} \approx 8{,}1$ cm.

58



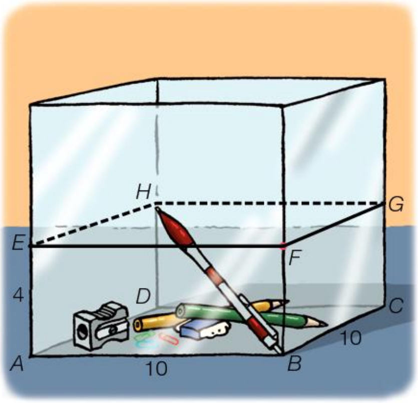
$KL^2 = KT^2 + TU^2 + UL^2 = 7^2 + 3^2 + 2^2 = 49 + 9 + 4 = 62$
Dus $KL = \sqrt{62} \approx 7{,}9$ cm.

59 **a** Zie de schets hiernaast.
$AP^2 = AR^2 + RS^2 + SP^2$
$= 5^2 + 4^2 + 3^2 = 50$
$BP^2 = BR^2 + RS^2 + SP^2$ en omdat $BR = AR$ is $BP^2 = AP^2$.
Dus $AP = BP = \sqrt{50} \approx 7{,}07$ cm.



b $\left.\begin{array}{l} AP^2 + BP^2 = 50 + 50 = 100 \\ AB^2 = 10^2 = 100 \end{array}\right\}$ $AP^2 + BP^2 = AB^2$, dus $\angle P = 90°$.

60 Zie de figuur hiernaast.
$BH^2 = AB^2 + AD^2 + DH^2 = 10^2 + 10^2 + 4^2 = 100 + 100 + 16 = 216$
$BP = \sqrt{216} \approx 14{,}7$ cm
De lengte van de pen is 147 mm.



61 Het is een kubus. Stel dat alle ribben x cm lang zijn. Dan krijg je
$x^2 + x^2 + x^2 = 12^2$
$3x^2 = 144$
: 3   : 3
$x^2 = 48$
$x = \sqrt{48} = 6{,}92...$ cm
Alle ribben zijn dus 6,92... cm.
inhoud kubus $= 6{,}92...^3 \approx 333$ cm^3

5.6 Stellingen in rechthoekige driehoeken

Bladzijde 33

62 **a** In $\triangle ABC$ is $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$

$$3^2 + 4^2 = AB^2$$
$$9 + 16 = AB^2$$
$$AB^2 = 25$$
$$AB = \sqrt{25} = 5 \text{ cm}$$

b opp $\triangle ABC = \frac{1}{2} \cdot AC \cdot BC = \frac{1}{2} \cdot 3 \cdot 4 = 6$ cm^2
opp $\triangle ABC = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 5 \cdot CD = 2{,}5 \cdot CD$ cm^2
Hieruit volgt $2{,}5 \cdot CD = 6$ ofwel $CD = \dfrac{6}{2{,}5} = 2{,}4$ cm.

Bladzijde 34

63 $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$

$$4^2 + 7^2 = AC^2$$
$$16 + 49 = AC^2$$
$$AC^2 = 65$$
$$AC = \sqrt{65} \text{ cm}$$

bach-stelling: $AB \cdot BC = AC \cdot BE$

$$7 \cdot 4 = \sqrt{65} \cdot BE$$
$$\sqrt{65} \cdot BE = 28$$
$$BE = \frac{28}{\sqrt{65}} \approx 3{,}5 \text{ cm}$$

64 $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$

$$6^2 + BC^2 = 10^2$$
$$36 + BC^2 = 100$$
$$BC^2 = 64$$
$$BC = \sqrt{64} = 8 \text{ cm}$$

bach-stelling: $AC \cdot BC = AB \cdot CD$

$$6 \cdot 8 = 10 \cdot CD$$
$$10 \cdot CD = 48$$
$$CD = \frac{48}{10} = 4{,}8 \text{ cm}$$

$\angle K = 90°$, dus $KL^2 + KM^2 = LM^2$
$$6^2 + 6^2 = LM^2$$
$$36 + 36 = LM^2$$
$$LM^2 = 72$$
$$LM = \sqrt{72} \text{ cm}$$
bach-stelling: $KL \cdot KM = LM \cdot KN$
$$6 \cdot 6 = \sqrt{72} \cdot KN$$
$$\sqrt{72} \cdot KN = 36$$
$$KN = \frac{36}{\sqrt{72}} \approx 4{,}2 \text{ cm}$$

65 $\angle S = 90°$, dus $PS^2 + RS^2 = PR^2$
$$50^2 + 20^2 = PR^2$$
$$2500 + 400 = PR^2$$
$$PR^2 = 2900$$
$$PR = \sqrt{2900} = 53{,}851... \text{ cm}$$
Stel het snijpunt van de latjes T.
bach-stelling: $PS \cdot RS = PR \cdot ST$
$$50 \cdot 20 = \sqrt{2900} \cdot ST$$
$$\sqrt{2900} \cdot ST = 1000$$
$$ST = \frac{1000}{\sqrt{2900}} \approx 18{,}569... \text{ cm}$$
De latjes QS en PR zijn samen $2 \cdot 18{,}569... + 53{,}851... \approx 91{,}0$ cm.

66
a $a^2 + b^2 = c^2$
b
$$\begin{array}{l} h^2 + p^2 = b^2 \\ h^2 + q^2 = a^2 \\ \hline 2h^2 + p^2 + q^2 = a^2 + b^2 \end{array} +$$
c $2h^2 + p^2 + q^2 = c^2$
d $c^2 = (p + q)^2 = p^2 + 2pq + q^2$
e Uit de vragen c en d volgt $2h^2 + p^2 + q^2 = p^2 + 2pq + q^2$
$$\begin{array}{l} 2h^2 + p^2 + q^2 = p^2 + 2pq + q^2 \\ \quad -p^2 - q^2 \quad -p^2 \quad -q^2 \\ 2h^2 = 2pq \\ :2 \quad :2 \\ h^2 = pq \end{array}$$

Bladzijde 35

67 *hpq*-stelling: $CD^2 = AD \cdot BD$
$$3{,}6^2 = AD \cdot 2{,}4$$
$$12{,}96 = AD \cdot 2{,}4$$
$$AD = \frac{12{,}96}{2{,}4} = 5{,}4 \text{ cm}$$
Dus $AB = 5{,}4 + 2{,}4 = 7{,}8$ cm en opp $\triangle ABC = \frac{1}{2} \cdot 7{,}8 \cdot 3{,}6 \approx 14{,}0 \text{ cm}^2$.

hpq-stelling in $\triangle PQS$: $PT^2 = QT \cdot ST$
$$PT^2 = 2{,}7 \cdot 2{,}5$$
$$PT^2 = 6{,}75$$
$$PT = \sqrt{6{,}75} \text{ cm}$$
opp $\triangle PQS = \frac{1}{2} \cdot 5{,}2 \cdot \sqrt{6{,}75} = 6{,}754... \text{ cm}^2$
hpq-stelling in $\triangle QRS$: $RU^2 = QU \cdot SU$
$$RU^2 = 1{,}6 \cdot 3{,}6$$
$$RU^2 = 5{,}76$$
$$PT = \sqrt{5{,}76} = 2{,}4 \text{ cm}$$
opp $\triangle QRS = \frac{1}{2} \cdot 5{,}2 \cdot 2{,}4 = 6{,}24 \text{ cm}^2$
opp $PQRS$ = opp $\triangle PQS$ + opp $\triangle QRS = 6{,}754... + 6{,}24 \approx 13{,}0 \text{ cm}^2$

68 Stel de hoogte van het dakdeel h.
Dan geldt volgens de *hpq*-stelling: $h^2 = (8 - 2) \cdot 2$
$h^2 = 6 \cdot 2$
$h^2 = 12$
$h = \sqrt{12} \approx 3{,}5$ m
Dus de hoogte van het huis is $5 + 3{,}5 = 8{,}5$ meter.

69 In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$
$6^2 + 8^2 = AC^2$
$36 + 64 = AC^2$
$AC^2 = 100$
$AC = \sqrt{100} = 10$ cm
bach-stelling in $\triangle ABC$: $AB \cdot BC = AC \cdot BE$
$6 \cdot 8 = 10 \cdot BE$
$10 \cdot BE = 48$
$BE = \dfrac{48}{10} = 4{,}8$ cm
In $\triangle ABE$ is $\angle E = 90°$, dus $AE^2 + BE^2 = AB^2$
$AE^2 + 4{,}8^2 = 6^2$
$AE^2 + 23{,}04 = 36$
$AE^2 = 12{,}96$
$AE = \sqrt{12{,}96} = 3{,}6$ cm
hpq-stelling in $\triangle ABD$: $AE^2 = BE \cdot DE$
$3{,}6^2 = 4{,}8 \cdot DE$
$12{,}96 = 4{,}8 \cdot DE$
$DE = \dfrac{12{,}96}{4{,}8} = 2{,}7$ cm
In $\triangle ADE$ is $\angle E = 90°$, dus $AE^2 + DE^2 = AD^2$
$3{,}6^2 + 2{,}7^2 = AD^2$
$12{,}96 + 7{,}29 = AD^2$
$AD^2 = 20{,}25$
$AD = \sqrt{20{,}25} = 4{,}5$ cm

Bladzijde 36

70 **a** Omdat het allemaal stralen zijn van de cirkel met middelpunt M.
$\triangle AMC$ en $\triangle BMC$ zijn allebei gelijkbenige driehoeken.
b $\angle A = \angle C_1$ (basishoeken)
$\angle B = \angle C_2$ (basishoeken)
c $\angle A + \angle B + \angle C_{12} = 180°$ (hoekensom driehoek)
Antwoord van vraag b gebruiken geeft
$\angle C_1 + \angle C_2 + \angle C_1 + \angle C_2 = 180°$
$2 \cdot \angle C_1 + 2 \cdot \angle C_2 = 180°$
d $2 \cdot \angle C_1 + 2 \cdot \angle C_2 = 180°$
: 2 **: 2** **: 2**
$\angle C_1 + \angle C_2 = 90°$
$\angle C_{12} = 90°$
Dus $\triangle ABC$ is een rechthoekige driehoek met $\angle C_{12} = 90°$.

71 AB is middellijn, dus volgens de stelling van Thales is $\angle C = 90°$.
De stelling van Pythagoras geeft $AC^2 + BC^2 = AB^2$
$7^2 + BC^2 = 8^2$
$49 + BC^2 = 64$
− 49 **− 49**
$BC^2 = 15$
$BC = \sqrt{15} = 3{,}87...$ cm
omtrek $\triangle ABC = 8 + 3{,}87... + 7 \approx 18{,}9$ cm

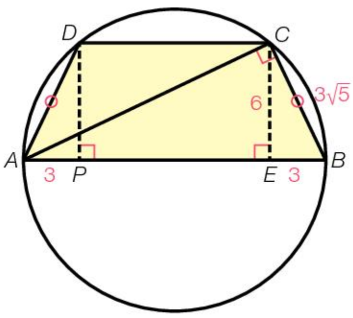
Bladzijde 37

72 PR is middellijn, dus volgens de stelling van Thales is $\angle S = 90°$ en $\angle Q = 90°$.
De stelling van Pythagoras in $\triangle PRS$ geeft $PS^2 + RS^2 = PR^2$
$$6^2 + 2^2 = PR^2$$
$$36 + 4 = PR^2$$
$$PR^2 = 40$$
$$PR = \sqrt{40}$$
De stelling van Pythagoras in $\triangle PQR$ geeft $PQ^2 + QR^2 = PR^2$
$$4^2 + QR^2 = 40$$
$$16 + QR^2 = 40$$
$$-16 \qquad -16$$
$$QR^2 = 24$$
$$QR = \sqrt{24}$$
opp $\triangle PQR = \frac{1}{2} \cdot 4 \cdot \sqrt{24} = 2\sqrt{24}$ en opp $\triangle PRS = \frac{1}{2} \cdot 2 \cdot 6 = 6$

opp $PQRS = 2\sqrt{24} + 6 \approx 15{,}80$ cm^2

73 PQ is middellijn, dus volgens de stelling van Thales is $\angle R = 90°$.
De stelling van Pythagoras in $\triangle PRQ$ geeft $PR^2 + QR^2 = PQ^2$
$$4^2 + 5^2 = PQ^2$$
$$16 + 25 = PQ^2$$
$$PQ^2 = 41$$
$$PQ = \sqrt{41} = 6{,}403... \text{ cm}$$
Dus $MQ = 6{,}403... : 2 = 3{,}201...$ cm
opp blauwe gebied = opp cirkel − opp $\triangle PQR = \pi \cdot 3{,}201...^2 - \frac{1}{2} \cdot 4 \cdot 5 \approx 22{,}20$ cm^2

74 Zie de figuur hiernaast.
AB is middellijn, dus $\triangle ABC$ is volgens de stelling van Thales rechthoekig met $\angle C = 90°$.
$BC = AD = 3\sqrt{5}$
De stelling van Pythagoras in $\triangle BCE$ geeft $BE^2 + CE^2 = BC^2$
$$3^2 + CE^2 = (3\sqrt{5})^2$$
$$9 + CE^2 = 45$$
$$-9 \qquad -9$$
$$CE^2 = 36$$
$$CE = \sqrt{36} = 6$$



hpq-stelling in $\triangle ABC$: $CE^2 = AE \cdot BE$
$$6^2 = AE \cdot 3$$
$$36 = AE \cdot 3$$
$$AE = \frac{36}{3} = 12 \text{ cm}$$
$AE = 12$ cm geeft $AB = 12 + 3 = 15$ cm en $CD = EP = 12 - 3 = 9$ cm

opp $ABCD = \frac{1}{2}(15 + 9) \cdot 6 = 72$ cm^2

Gemengde opgaven

Bladzijde 38

1 In $\triangle ADE$ is $\angle D = 90°$, dus $AD^2 + DE^2 = AE^2$
$$AD^2 + 3^2 = 5^2$$
$$AD^2 + 9 = 25$$
$$-9 \quad -9$$
$$AD^2 = 16$$
In $\triangle ABD$ is $\angle A = 90°$, dus $AB^2 + AD^2 = BD^2$
$$2^2 + 16 = BD^2$$
$$4 + 16 = BD^2$$
$$BD^2 = 20$$

In $\triangle BCD$ is $\angle D = 90°$, dus $BD^2 + CD^2 = BC^2$

$$\begin{aligned}
20 + CD^2 &= 5^2 \\
20 + CD^2 &= 25 \\
-20 \qquad &\quad -20 \\
CD^2 &= 5 \\
CD &= \sqrt{5} \approx 2{,}2 \text{ cm}
\end{aligned}$$

In $\triangle FGH$ is $\angle G = 90°$, dus $FG^2 + GH^2 = FH^2$

$$\begin{aligned}
4{,}1^2 + 2{,}9^2 &= FH^2 \\
16{,}81 + 8{,}41 &= FH^2 \\
FH^2 &= 25{,}22 \\
FH &= \sqrt{25{,}22} \text{ cm}
\end{aligned}$$

In $\triangle FHI$ is $\angle I = 90°$, dus $FI^2 + HI^2 = FH^2$

$$\begin{aligned}
FI^2 + 4{,}5^2 &= 25{,}22 \\
-20{,}25 \qquad &\quad -20{,}25 \\
FI^2 &= 4{,}97 \\
FI &= \sqrt{4{,}97} \text{ cm}
\end{aligned}$$

bach-stelling in $\triangle FHI$: $FI \cdot HI = FH \cdot IK$

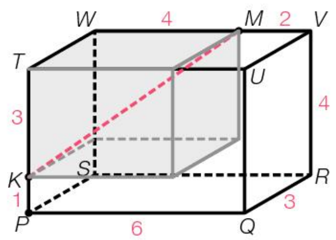
$$\begin{aligned}
\sqrt{4{,}97} \cdot 4{,}5 &= \sqrt{25{,}22} \cdot IK \\
IK &= \frac{4{,}5\sqrt{4{,}97}}{\sqrt{25{,}22}} \approx 2{,}0 \text{ cm}
\end{aligned}$$

2 In $\triangle CFE$ is $\angle F = 90°$, dus $CF^2 + EF^2 = CE^2$

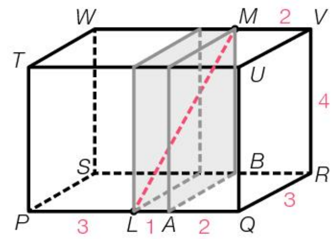
$$\begin{aligned}
5^2 + 4^2 &= CE^2 \\
25 + 16 &= CE^2 \\
CE^2 &= 41
\end{aligned}$$

Dus $CE = \sqrt{41} \approx 6{,}4$ cm.

Zie de figuur hiernaast.
$KM^2 = KT^2 + TW^2 + WM^2 = 3^2 + 3^2 + 4^2 = 9 + 9 + 16 = 34$
Dus $KM = \sqrt{34} \approx 5{,}8$ cm.

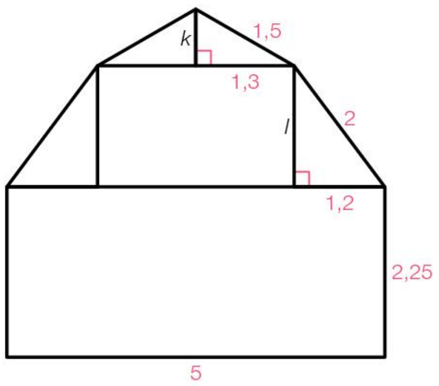


Zie de figuur hiernaast.
$LM^2 = LA^2 + AB^2 + BM^2 = 1^2 + 3^2 + 4^2 = 1 + 9 + 16 = 26$
Dus $LM = 26 \approx 5{,}1$ cm.



3 Zie de schets hiernaast.
Van de rechthoekige driehoek rechtsboven is de langste rechthoekszijde $2{,}6 : 2 = 1{,}3$ m.
Er geldt $1{,}3^2 + k^2 = 1{,}5^2$

$$\begin{aligned}
1{,}69 + k^2 &= 2{,}25 \\
-1{,}69 \qquad &\quad -1{,}69 \\
k^2 &= 0{,}56 \\
k &= \sqrt{0{,}56} = 0{,}748\ldots
\end{aligned}$$

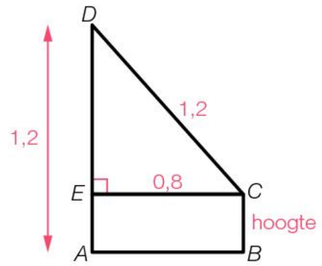


Van de rechthoekige driehoek rechtsonder is de korste rechthoekszijde $\frac{(5-2{,}6)}{2} = 1{,}2$ m.
Er geldt $1{,}2^2 + l^2 = 2^2$
$1{,}44 + l^2 = 4$
$-1{,}44 \quad -1{,}44$
$l^2 = 2{,}56$
$l = \sqrt{2{,}56} = 1{,}6$ m
De hoogte van het huis is $2{,}25 + 0{,}748... + 1{,}6 \approx 4{,}60$ m.

4 **a** Het punt recht onder B noemen we D en het punt rechts vooraan noemen we C.
$BD = 350$ m $= 0{,}35$ km
AB is de lengte van een lichaamsdiagonaal in de balk met lengte 8,5 km, breedte 3,2 km en hoogte 0,35 km.
$AB^2 = 8{,}5^2 + 3{,}2^2 + 0{,}35^2 = 82{,}6125$
$AB = \sqrt{82{,}6125} = 9{,}0891...$ km
Dus de lengte van Jordy's route is 9089 meter.

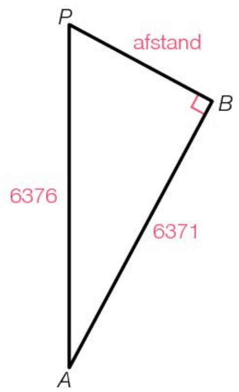
b 3 km/uur, dus 9,089 km in $\frac{9{,}089}{3} \approx 3{,}03$ uur.
Dat is 3 uur en $0{,}03 \cdot 60 = 1{,}8$ minuten.
Zij doet er dus ruim drie uur over.

5 Zie de figuur hiernaast.
In $\triangle CDE$ is $\angle E = 90°$, dus $CE^2 + DE^2 = CD^2$
$0{,}8^2 + DE^2 = 1{,}2^2$
$0{,}64 + DE^2 = 1{,}44$
$-0{,}64 \quad -0{,}64$
$DE^2 = 0{,}8$
$DE = \sqrt{0{,}8} = 0{,}894...$ m
De hoogte is $1{,}20 - 0{,}894... = 0{,}305...$ m ≈ 31 cm.

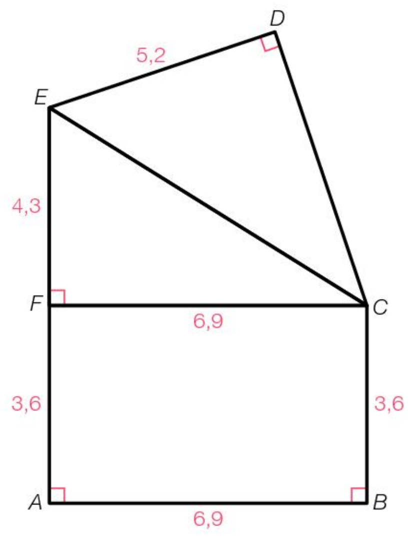


Bladzijde 39

6 Zie de figuur hiernaast.
In $\triangle ABP$ is $\angle B = 90°$, dus $AB^2 + BP^2 = AP^2$
$6371^2 + BP^2 = (6371 + 5)^2$
$40\,589\,641 + BP^2 = 40\,653\,376$
$-40\,589\,641 \quad -40\,589\,641$
$BP^2 = 63\,735$
$BP = \sqrt{63\,735} \approx 252$ km
De afstand is 252 km.



7 $EF = AE - AF = 7{,}9 - 3{,}6 = 4{,}3$ cm
In $\triangle CEF$ is $\angle F = 90°$, dus $CF^2 + EF^2 = CE^2$
$6{,}9^2 + 4{,}3^2 = CE^2$
$47{,}61 + 18{,}49 = CE^2$
$CE^2 = 66{,}1$
In $\triangle CDE$ is $\angle D = 90°$, dus $CD^2 + DE^2 = CE^2$
$CD^2 + 5{,}2^2 = 66{,}1$
$CD^2 + 27{,}04 = 66{,}1$
$-27{,}04 \quad -27{,}04$
$CD^2 = 39{,}06$
$CD = \sqrt{39{,}06} \approx 6{,}2$ cm



8 **a** Zie de figuur hiernaast.

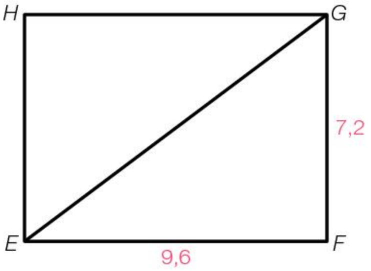
In $\triangle EFG$ is $\angle F = 90°$, dus $EF^2 + FG^2 = EG^2$

$9{,}6^2 + 7{,}2^2 = EG^2$

$92{,}16 + 51{,}84 = EG^2$

$EG^2 = 144$

$EG = \sqrt{144} = 12$ m



b Zie de figuur hiernaast.

$ES = 12 : 2 = 6$m

In $\triangle EST$ is $\angle S = 90°$, dus $ES^2 + ST^2 = ET^2$

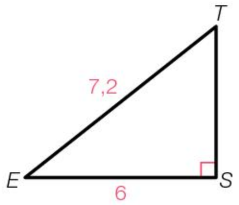
$6^2 + ST^2 = 7{,}2^2$

$36 + ST^2 = 51{,}84$

$-36 \qquad -36$

$ST^2 = 15{,}84$

$ST = \sqrt{15{,}84} = 3{,}97...$ m



De hoogte van het huis is $3 + 3{,}97... \approx 7{,}0$ meter.

c Zie de figuur hiernaast.

In $\triangle FUT$ is $\angle U = 90°$, dus $FU^2 + TU^2 = FT^2$

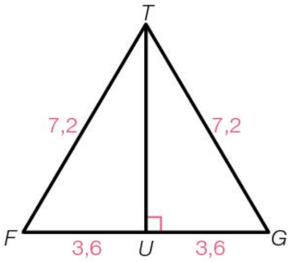
$3{,}6^2 + TU^2 = 7{,}2^2$

$12{,}96 + TU^2 = 51{,}84$

$-12{,}96 \qquad -12{,}96$

$TU^2 = 38{,}88$

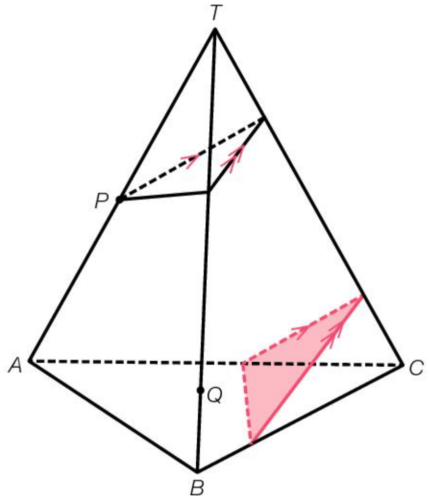
$TU = \sqrt{38{,}88} = 6{,}235...$



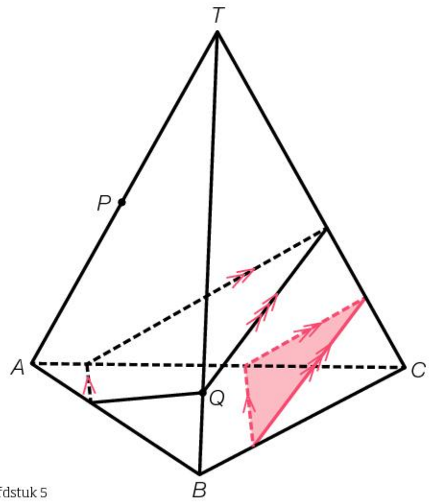
opp $\triangle TFG = \frac{1}{2} \cdot 7{,}2 \cdot 6{,}235... \approx 22{,}4$ m^2

d $\left.\begin{array}{l} ET^2 + GT^2 = 7{,}2^2 + 7{,}2^2 = 103{,}68 \\ EG^2 = 12^2 = 144 \end{array}\right\}$ $ET^2 + GT^2 \neq EG^2$, dus $\triangle EGT$ is niet rechthoekig.

9 **a**



b



10 In $\triangle LMQ$ is $\angle Q = 90°$, dus $LQ^2 + MQ^2 = LM^2$

$$\begin{aligned} LQ^2 + 2{,}1^2 &= 3^2 \\ LQ^2 + 4{,}41 &= 9 \\ -\,4{,}41 \quad & \quad -\,4{,}41 \\ LQ^2 &= 4{,}59 \\ LQ &= \sqrt{4{,}59}\text{ cm} \end{aligned}$$

In $\triangle MNQ$ is $\angle Q = 90°$, dus $MQ^2 + NQ^2 = MN^2$

$$\begin{aligned} 2{,}1^2 + NQ^2 &= 5^2 \\ 4{,}41 + NQ^2 &= 25 \\ -\,4{,}41 \quad & \quad -\,4{,}41 \\ NQ^2 &= 20{,}59 \\ NQ &= \sqrt{20{,}59}\text{ cm} \end{aligned}$$

hpq-stelling in $\triangle LNP$: $PQ^2 = LQ \cdot NQ$

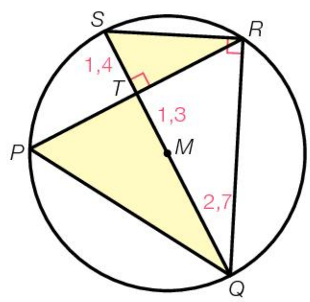
$$\begin{aligned} PQ^2 &= \sqrt{4{,}59} \cdot \sqrt{20{,}59} \\ PQ^2 &= 9{,}7215... \\ PQ &= \sqrt{9{,}7215...} \approx 3{,}1\text{ cm} \end{aligned}$$

11 Zie de schets hiernaast.
QS is middellijn, dus volgens de stelling van Thales is $\angle R = 90°$.
hpq-stelling in $\triangle QRS$: $RT^2 = ST \cdot QT$

$$RT^2 = 1{,}4 \cdot 4 = 5{,}6$$

In $\triangle TRS$ is $\angle T = 90°$, dus $TS^2 + RT^2 = RS^2$

$$\begin{aligned} 1{,}4^2 + 5{,}6 &= RS^2 \\ 1{,}96 + 5{,}6 &= RS^2 \\ RS^2 &= 7{,}56 \\ RS &= \sqrt{7{,}56} \approx 2{,}7\text{ cm} \end{aligned}$$



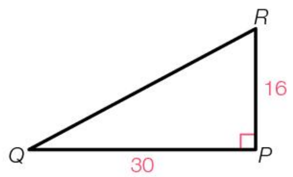
Diagnostische toets

Bladzijde 42

1 $\triangle ABC$: $AB^2 + BC^2 = AC^2$
$\triangle ABD$: $AD^2 + BD^2 = AB^2$
$\triangle BCD$: $BD^2 + CD^2 = BC^2$

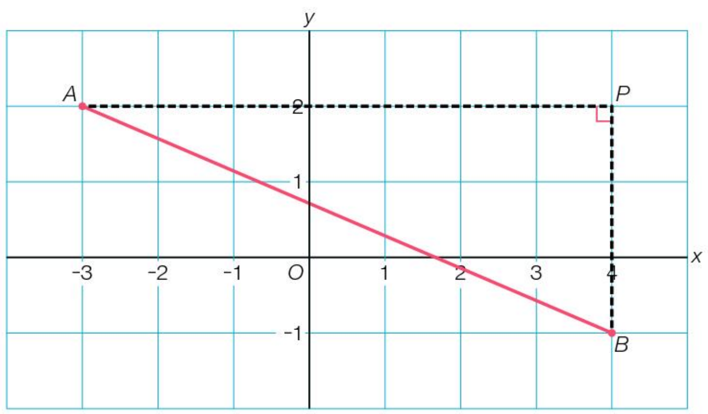
2 Zie de schets hiernaast.
$\angle P = 90°$, dus $PQ^2 + PR^2 = QR^2$

$$\begin{aligned} 30^2 + 16^2 &= QR^2 \\ 900 + 256 &= QR^2 \\ QR^2 &= 1156 \\ QR &= \sqrt{1156} = 34\text{ cm} \end{aligned}$$



3 Zie de figuur hiernaast.
$\angle P = 90°$, dus $AP^2 + BP^2 = AB^2$

$$\begin{aligned} 7^2 + 3^2 &= AB^2 \\ 49 + 9 &= AB^2 \\ AB^2 &= 58 \\ AB &= \sqrt{58} \approx 7{,}62 \end{aligned}$$



4 Zie de schets hiernaast.

$\angle P = 90°$, dus $PQ^2 + PR^2 = QR^2$

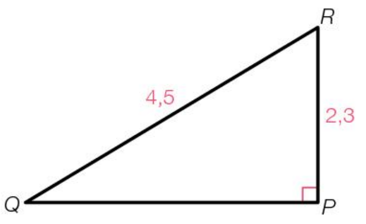
$PQ^2 + 2{,}3^2 = 4{,}5^2$

$PQ^2 + 5{,}29 = 20{,}25$

$- 5{,}29 \quad - 5{,}29$

$PQ^2 = 14{,}96$

$PQ = \sqrt{14{,}96} \approx 3{,}9$ cm



5 In $\triangle ADC$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$

$35^2 + 12^2 = AC^2$

$1225 + 144 = AC^2$

$AC^2 = 1369$

In $\triangle BCD$ is $\angle D = 90°$, dus $BD^2 + CD^2 = BC^2$

$5^2 + 12^2 = BC^2$

$25 + 144 = BC^2$

$BC^2 = 169$

$$\left.\begin{array}{l} AC^2 + BC^2 = 1369 + 169 = 1538 \\ AB^2 = (35 + 5)^2 = 1600 \end{array}\right\} AC^2 + BC^2 \neq AB^2, \text{ dus } \triangle ABC \text{ is niet rechthoekig.}$$

6 Zie de schets hiernaast.

In $\triangle ADE$ is $\angle E = 90°$, dus $AE^2 + DE^2 = AD^2$

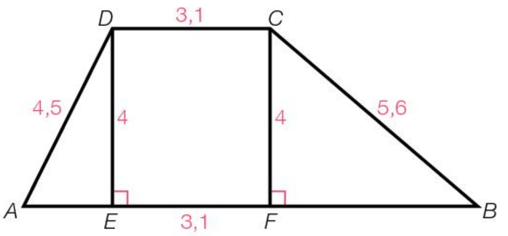
$AE^2 + 4^2 = 4{,}5^2$

$AE^2 + 16 = 20{,}25$

$- 16 \quad - 16$

$AE^2 = 4{,}25$

$AE = \sqrt{4{,}25} = 2{,}06...$ cm



In $\triangle BCF$ is $\angle F = 90°$, dus $BF^2 + CF^2 = BC^2$

$BF^2 + 4^2 = 5{,}6^2$

$BF^2 + 16 = 31{,}36$

$- 16 \quad - 16$

$BF^2 = 15{,}36$

$BF = \sqrt{15{,}36} = 3{,}91...$ cm

$AB = AE + EF + BF = 2{,}06... + 3{,}1 + 3{,}91... \approx 9{,}1$ cm

7 Zie de schets hiernaast.

In $\triangle ADC$ is $\angle D = 90°$, dus $AD^2 + CD^2 = AC^2$

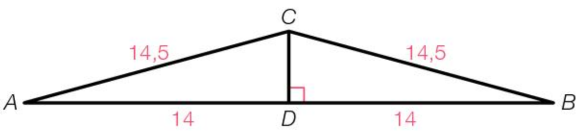
$14^2 + CD^2 = 14{,}5^2$

$196 + CD^2 = 210{,}25$

$- 196 \quad - 196$

$CD^2 = 14{,}25$

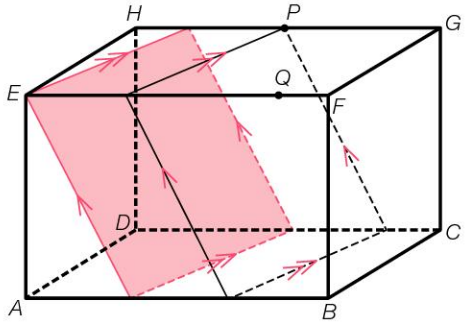
$CD = \sqrt{14{,}25} = 3{,}77...$ m



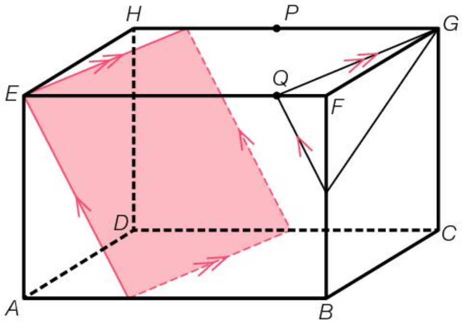
De hoogte van de hangar is $4{,}5 + 3{,}77... \approx 8{,}3$ m.

Bladzijde 43

8 **a**

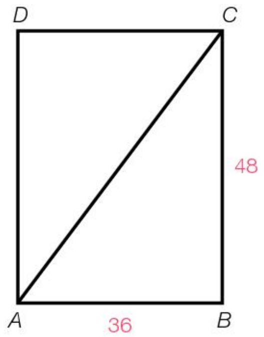


b



9 Zie de schets hiernaast van het grondvlak $ABCD$.
In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

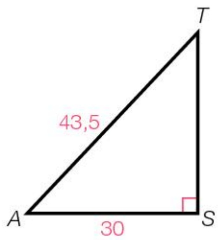
$$36^2 + 48^2 = AC^2$$
$$1296 + 2304 = AC^2$$
$$AC^2 = 3600$$
$$AC = \sqrt{3600} = 60 \text{ cm}$$



Zie de schets hiernaast.
AS is de helft van AC, dus $AS = 60 : 2 = 30$ cm.
In $\triangle AST$ is $\angle S = 90°$, dus $AS^2 + ST^2 = AT^2$

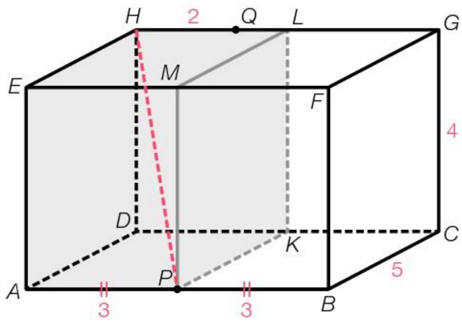
$$30^2 + ST^2 = 43{,}5^2$$
$$900 + ST^2 = 1892{,}25$$
$$-900 \qquad -900$$
$$ST^2 = 992{,}25$$
$$ST = \sqrt{992{,}25} = 31{,}5 \text{ cm}$$

De hoogte ST is 31,5 cm.

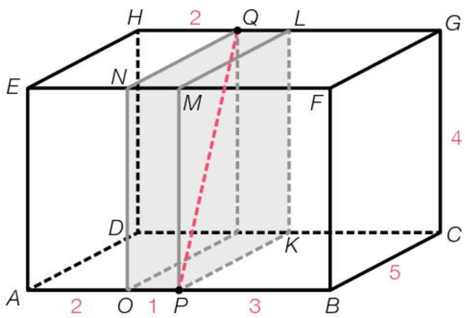


10 **a** $CE^2 = CB^2 + BA^2 + AE^2 = 5^2 + 6^2 + 4^2 = 25 + 36 + 16 = 77$
Dus $CE = \sqrt{77} \approx 8{,}8$ cm.

b Zie de figuur hiernaast.
$HP^2 = HE^2 + EA^2 + AP^2 = 5^2 + 4^2 + 3^2 = 25 + 16 + 9 = 50$
Dus $HP = \sqrt{50} \approx 7{,}1$ cm.



c Zie de figuur hiernaast.
$PQ^2 = PK^2 + KL^2 + LQ^2 = 5^2 + 4^2 + 1^2 = 25 + 16 + 1 = 42$
Dus $PQ = \sqrt{42} \approx 6{,}5$ cm.



11 In $\triangle PQR$ is $\angle P = 90°$, dus $PQ^2 + PR^2 = QR^2$

$$9^2 + 5^2 = QR^2$$
$$81 + 25 = QR^2$$
$$QR^2 = 106$$
$$QR = \sqrt{106} \text{ cm}$$

bach-stelling: $PQ \cdot PR = QR \cdot PS$

$$9 \cdot 5 = \sqrt{106} \cdot PS$$
$$\sqrt{106} \cdot PS = 45$$
$$PS = \frac{45}{\sqrt{106}} \approx 4{,}4 \text{ cm}$$

12 *hpq*-stelling: $KN^2 = LN \cdot MN$

$2{,}1^2 = 1{,}2 \cdot MN$

$4{,}41 = 1{,}2 \cdot MN$

$MN = \frac{4{,}41}{1{,}2} \approx 3{,}7$ cm

13 PQ is middellijn, dus volgens de stelling van Thales is $\angle R = 90°$.
De stelling van Pythagoras in $\triangle PRQ$ geeft $PR^2 + RQ^2 = PQ^2$

$2^2 + 3^2 = PQ^2$

$4 + 9 = PQ^2$

$PQ^2 = 13$

$PQ = \sqrt{13} = 3{,}605...$

$PM = 3{,}605... : 2 = 1{,}802...$

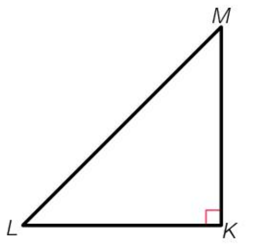
opp blauwe gebied = opp cirkel − opp $\triangle PQR = \pi \cdot 1{,}802...^2 - \frac{1}{2} \cdot 2 \cdot 3 \approx 7{,}21$ cm^2

Herhaling

Bladzijde 44

1 $\triangle QRS$: $QS^2 + RS^2 = QR^2$
$\triangle PQR$: $PR^2 + QR^2 = PQ^2$

2 **a** Zie de schets hiernaast.
$\angle K = 90°$, dus $KL^2 + KM^2 = LM^2$



b $KL^2 + KM^2 = LM^2$

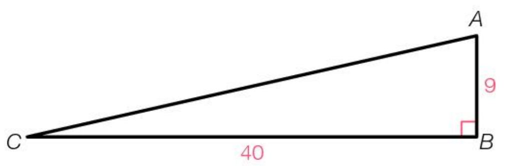
$20^2 + 21^2 = LM^2$

$400 + 441 = LM^2$

$LM^2 = 841$

$LM = \sqrt{841} = 29$ cm

3 Zie de schets hiernaast.
$\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$



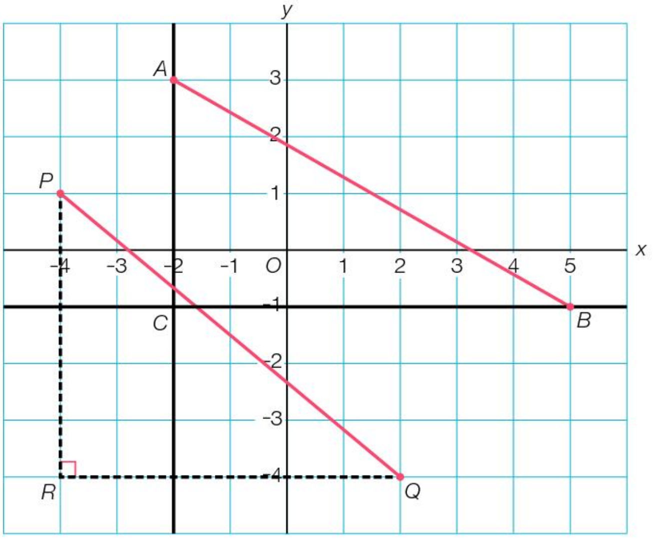
$9^2 + 40^2 = AC^2$

$81 + 1600 = AC^2$

$AC^2 = 1681$

$AC = \sqrt{1681} = 41$ cm

4 **a,b,f**



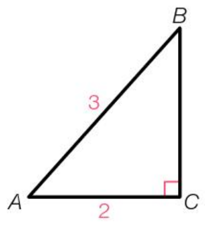
c $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$

d $BC = 7$

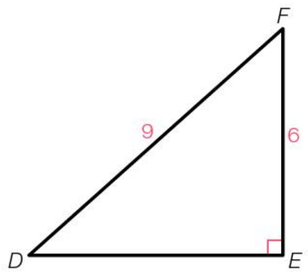
e $AC^2 + BC^2 = AB^2$
$4^2 + 7^2 = AB^2$
$16 + 49 = AB^2$
$AB^2 = 65$
$AB = \sqrt{65} \approx 8{,}06$

f $\angle R = 90°$, dus $PR^2 + QR^2 = PQ^2$
$5^2 + 6^2 = PQ^2$
$25 + 36 = PQ^2$
$PQ^2 = 61$
$PQ = \sqrt{61} \approx 7{,}81$

5 Zie de schets hiernaast.
$\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$
$2^2 + BC^2 = 3^2$
$4 + BC^2 = 9$
$-4 \qquad -4$
$BC^2 = 5$
$BC = \sqrt{5} \approx 2{,}2$ cm



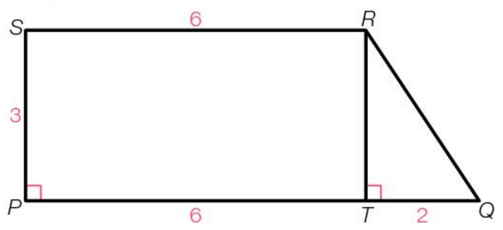
6 Zie de schets hiernaast.
$\angle E = 90°$, dus $DE^2 + EF^2 = DF^2$
$DE^2 + 6^2 = 9^2$
$DE^2 + 36 = 81$
$-36 \qquad -36$
$DE^2 = 45$
$DE = \sqrt{45} \approx 6{,}71$ cm



7 **a** $KM^2 + LM^2 = 28^2 + 45^2 = 2809$
b $KL^2 = 53^2 = 2809$
c $KM^2 + LM^2 = KL^2$, dus $\angle M = 90°$
Dus $\triangle KLM$ is rechthoekig.

Bladzijde 45

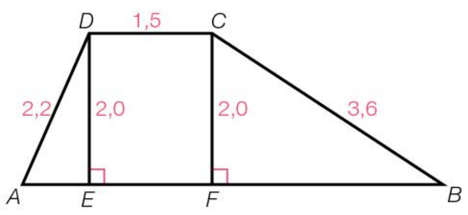
8 **a**



b In $\triangle QRT$ is $\angle T = 90°$, dus $TQ^2 + TR^2 = QR^2$
$2^2 + 3^2 = QR^2$
$4 + 9 = QR^2$
$QR^2 = 13$
$QR = \sqrt{13} = 3{,}60... \approx 3{,}6$ cm

c omtrek trapezium $PQRS = 8 + 3 + 6 + 3{,}60... \approx 20{,}6$ cm

9 **a,b** Zie de schets hiernaast.
In $\triangle ADE$ is $\angle E = 90°$, dus $AE^2 + DE^2 = AD^2$
$AE^2 + 2{,}0^2 = 2{,}2^2$
$AE^2 + 4 = 4{,}84$
$-4 \qquad -4$
$AE^2 = 0{,}84$
$AE = \sqrt{0{,}84} = 0{,}916... \approx 0{,}92$ cm



c In $\triangle BCF$ is $\angle F = 90°$, dus $BF^2 + CF^2 = BC^2$

$$BF^2 + 2{,}0^2 = 3{,}6^2$$
$$BF^2 + 4 = 12{,}96$$
$$-4 \quad -4$$
$$BF^2 = 8{,}96$$
$$BF = \sqrt{8{,}96} = 2{,}99... \text{ cm}$$

$AB = AE + EF + BF = 0{,}916... + 1{,}5 + 2{,}99... \approx 5{,}4$ cm

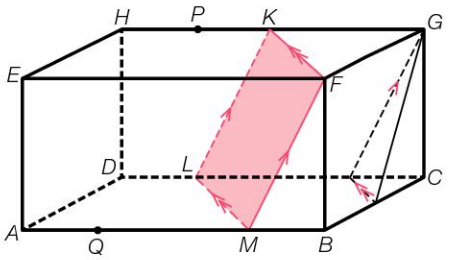
10 **a** $EF = CF = 5 : 2 = 2{,}5$ m

$\angle F = 90°$, dus $DF^2 + EF^2 = DE^2$

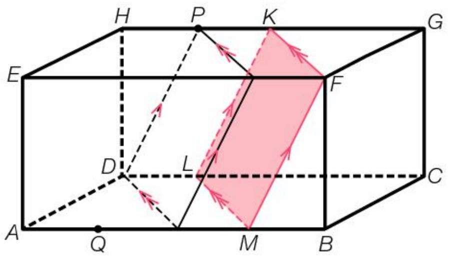
$$DF^2 + 2{,}5^2 = 3{,}8^2$$
$$DF^2 + 6{,}25 = 14{,}44$$
$$-6{,}25 \quad -6{,}25$$
$$DF^2 = 8{,}19$$
$$DF = \sqrt{8{,}19} = 2{,}861... \approx 2{,}86 \text{ m}$$

b De hoogte van het huis is $6 + 2{,}861... \approx 8{,}86$ m = 886 cm.

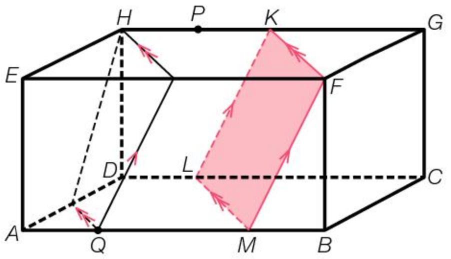
11 **a**



b

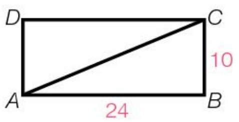


c



Bladzijde 46

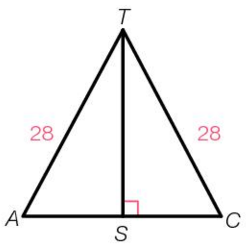
12 **a** Zie de schets hiernaast.



In $\triangle ABC$ is $\angle B = 90°$, dus $AB^2 + BC^2 = AC^2$

$$24^2 + 10^2 = AC^2$$
$$576 + 100 = AC^2$$
$$AC^2 = 676$$
$$AC = \sqrt{676} = 26 \text{ cm}$$

b



c Je weet $AT = 28$ cm en $AS = 26 : 2 = 13$ cm.

d In $\triangle AST$ is $\angle S = 90°$, dus $AS^2 + ST^2 = AT^2$

$$13^2 + ST^2 = 28^2$$
$$169 + ST^2 = 784$$
$$-169 \qquad -169$$
$$ST^2 = 615$$
$$ST = \sqrt{615} \approx 24{,}8 \text{ cm}$$

13 **a** $AG^2 = AB^2 + BC^2 + CG^2$

b $AG^2 = AB^2 + BC^2 + CG^2 = 7^2 + 5^2 + 3^2 = 83$

Dus $AG = \sqrt{83} \approx 9{,}1$ cm.

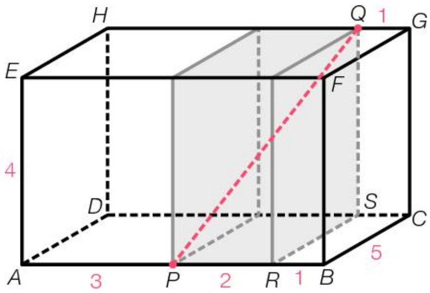
14 **a** $HP^2 = HD^2 + DA^2 + AP^2 = 4^2 + 5^2 + 3^2 = 50$

Dus $HP = \sqrt{50} \approx 7{,}1$ cm.

b Zie de schets hiernaast.

$PQ^2 = PR^2 + RS^2 + SQ^2 = 2^2 + 5^2 + 4^2 = 45$

Dus $PQ = \sqrt{45} \approx 6{,}7$ cm.



15 **a** In $\triangle PQR$ is $\angle P = 90°$, dus $PQ^2 + PR^2 = QR^2$

$$12^2 + 5^2 = QR^2$$
$$144 + 25 = QR^2$$
$$QR^2 = 169$$
$$QR = \sqrt{169} = 13 \text{ cm}$$

bach-stelling: $PQ \cdot PR = QR \cdot PS$

$$12 \cdot 5 = 13 \cdot PS$$
$$13 \cdot PS = 60$$
$$PS = \frac{60}{13} \approx 4{,}6 \text{ cm}$$

Bladzijde 47

b In $\triangle ABC$ is $\angle C = 90°$, dus $AC^2 + BC^2 = AB^2$

$$5^2 + 10^2 = AB^2$$
$$25 + 100 = AB^2$$
$$AB^2 = 125$$
$$AB = \sqrt{125} \text{ cm}$$

bach-stelling: $AC \cdot BC = AB \cdot CD$

$$5 \cdot 10 = \sqrt{125} \cdot CD$$
$$\sqrt{125} \cdot CD = 50$$
$$CD = \frac{50}{\sqrt{125}} \approx 4{,}5 \text{ cm}$$

16 **a** *hpq*-stelling: $MN^2 = KN \cdot LN$

$$4{,}2^2 = KN \cdot 1{,}8$$
$$17{,}64 = KN \cdot 1{,}8$$
$$KN = \frac{17{,}64}{1{,}8} = 9{,}8 \text{ cm}$$

b *hpq*-stelling: $FH^2 = EH \cdot GH$

$$FH^2 = 4 \cdot 6$$
$$FH^2 = 24$$
$$FH = \sqrt{24} \approx 4{,}9 \text{ cm}$$

17 **a** AB is middellijn, dus volgens de stelling van Thales is $\angle C = 90°$.
De stelling van Pythagoras in $\triangle ABC$ geeft $AC^2 + BC^2 = AB^2$
$2{,}4^2 + 4{,}5^2 = AB^2$
$5{,}76 + 20{,}25 = AB^2$
$AB^2 = 26{,}01$
$AB = \sqrt{26{,}01} = 5{,}1$ cm

b BC is middellijn, dus volgens de stelling van Thales is $\angle A = 90°$.
De stelling van Pythagoras in $\triangle ABC$ geeft $AB^2 + AC^2 = BC^2$
$2{,}7^2 + 4^2 = BC^2$
$7{,}29 + 16 = BC^2$
$BC^2 = 23{,}29$
$BC = \sqrt{23{,}29} = 4{,}82...$ cm
$BM = 4{,}82... : 2 = 2{,}41...$
opp cirkel $= \pi \cdot 2{,}41...^2 \approx 18{,}29$ cm^2

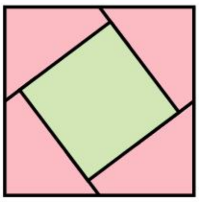
c EG is middellijn dus volgens de stelling van Thales is $\angle F = 90°$ in $\triangle EFG$.
hpq-stelling: $FH^2 = GH \cdot EH$
$1{,}7^2 = 0{,}9 \cdot EH$
$EH = \dfrac{2{,}89}{0{,}9} = 3{,}21...$ cm
De middellijn $EG = 3{,}21... + 0{,}9 = 4{,}11...$ cm.
De straal van de cirkel $= 4{,}11... : 2 = 2{,}05...$
opp blauwe gebied = opp cirkel − opp $\triangle EFG = \pi \cdot 2{,}05...^2 - \frac{1}{2} \cdot 4{,}11... \cdot 1{,}7 \approx 9{,}78$ cm^2

Onderzoek De stelling van Pythagoras bewijzen

Bladzijde 48

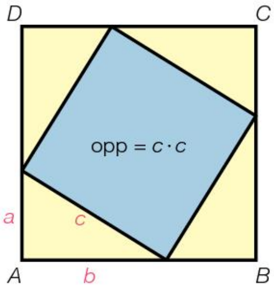
1 **a** *

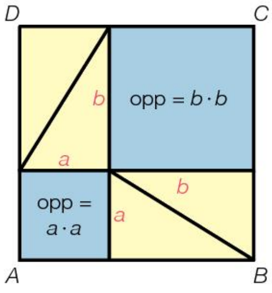
b



c De groene en rode stukken passen precies op het blauwe stuk.
Daaruit volgt dat $a \cdot a + b \cdot b = c \cdot c$ ofwel $a^2 + b^2 = c^2$.

2



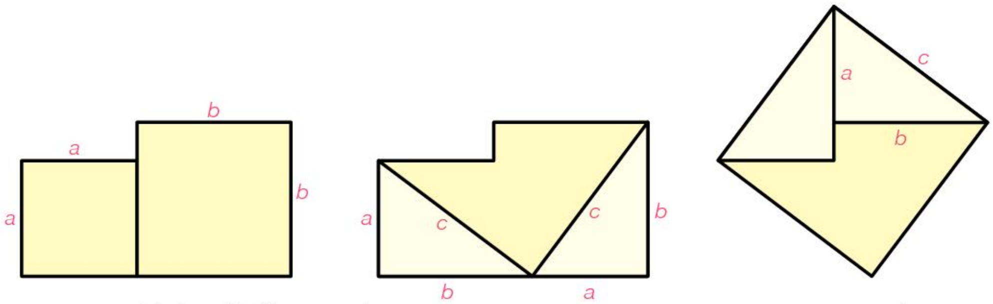


In figuur a zie je dat het blauwe gedeelte een oppervlakte heeft van $c \cdot c = c^2$.
In figuur b zie je dat het blauwe gedeelte een oppervlakte heeft van $a \cdot a + b \cdot b = a^2 + b^2$.
In beide grote vierkanten zitten dezelfde vier gele driehoeken, dus de blauwe oppervlakte in de rechter figuur is gelijk aan de blauwe oppervlakte in de linker figuur.
Hiermee heb je bewezen dat $a^2 + b^2 = c^2$.

Bladzijde 49

3 **a** *

b



a opp = $a \cdot a + b \cdot b = a^2 + b^2$ **b** **c** opp = $c \cdot c = c^2$

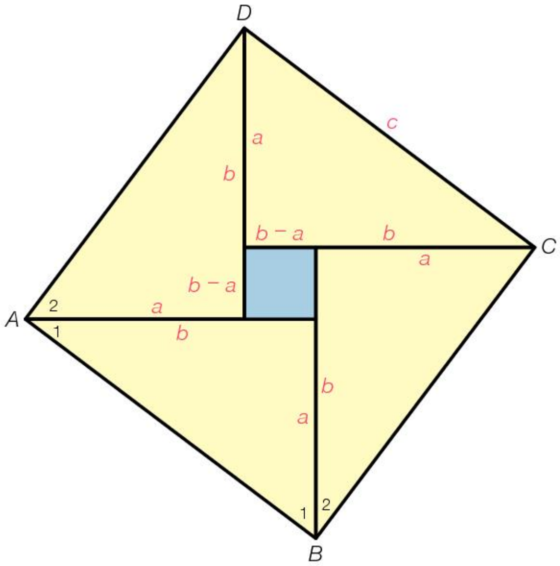
In figuur a is de totale oppervlakte van het gele gebied $a \cdot a + b \cdot b = a^2 + b^2$.
In de rechthoekige driehoeken met zijden a, b en c zijn de twee scherpe hoeken samen 90°. Dus in figuur c is elke hoek van de grote vierhoek 90°. En omdat de zijden van deze vierhoek elk een lengte c hebben, is deze vierhoek een vierkant met oppervlakte $c \cdot c = c^2$.
Hiermee heb je bewezen dat $a^2 + b^2 = c^2$.

4 **a** $\angle A = 90°$
In $\triangle AKN$ is $\angle A + \angle K_1 + \angle N_1 = 180°$.
Hieruit volgt dat $\angle K_1 + \angle N_1 = 180° - \angle A = 180° - 90° = 90°$.
$\angle A = \angle B = \angle C = \angle D = 90°$
Verder weet je dat $\angle N_1 = \angle K_3$, dus $\angle K_2 = 180° - 90° = 90°$.
Op dezelfde manier vind je dat $\angle N_2 = \angle L = \angle M = 90°$.
Alle zijden van $KLMN$ hebben lengte c.
Dus $KLMN$ is een vierkant.

b opp $ABCD$ = opp gele driehoeken + opp blauwe vierkant = $4 \cdot$ opp $\triangle AKN$ + opp $KLMN$
opp $\triangle AKN = \frac{1}{2} \cdot a \cdot b = \frac{1}{2}ab$
opp $KLMN = c \cdot c = c^2$
opp $ABCD = 4 \cdot$ opp $\triangle AKN$ + opp $KLMN$
$(a + b) \cdot (a + b) = 4 \cdot \frac{1}{2}ab + c^2$
$a^2 + 2ab + b^2 = 2ab + c^2$
$\quad - 2ab \qquad - 2ab$
$a^2 + b^2 = c^2$

5 **a** $\angle A_1 + \angle B_1 = 90°$ Verder weet je dat $\angle A_2 = \angle B_1$ en hieruit volgt dat $\angle A_1 + \angle A_2 = 90°$, dus $\angle A_{12} = 90°$.
Op dezelfde manier kun je aantonen dat $\angle B_{12}$, $\angle C_{12}$ en $\angle D_{12}$ ook 90° zijn.
Alle zijden van vierhoek $ABCD$ hebben lengte c, dus $ABCD$ is een vierkant.

b



Het kleine vierkant heeft zijden van $b - a$.
opp kleine vierkant = $(b - a) \cdot (b - a) = b^2 - 2ab + a^2 = a^2 - 2ab + b^2$

c opp $ABCD = c \cdot c = c^2$

opp elke gele driehoek $= \frac{1}{2} \cdot a \cdot b = \frac{1}{2}ab$

opp $ABCD = 4 \cdot$ opp gele driehoek + opp blauwe vierkant

$c^2 = 4 \cdot \frac{1}{2}ab + a^2 - 2ab + b^2 = a^2 + b^2$

Dus $a^2 + b^2 = c^2$.